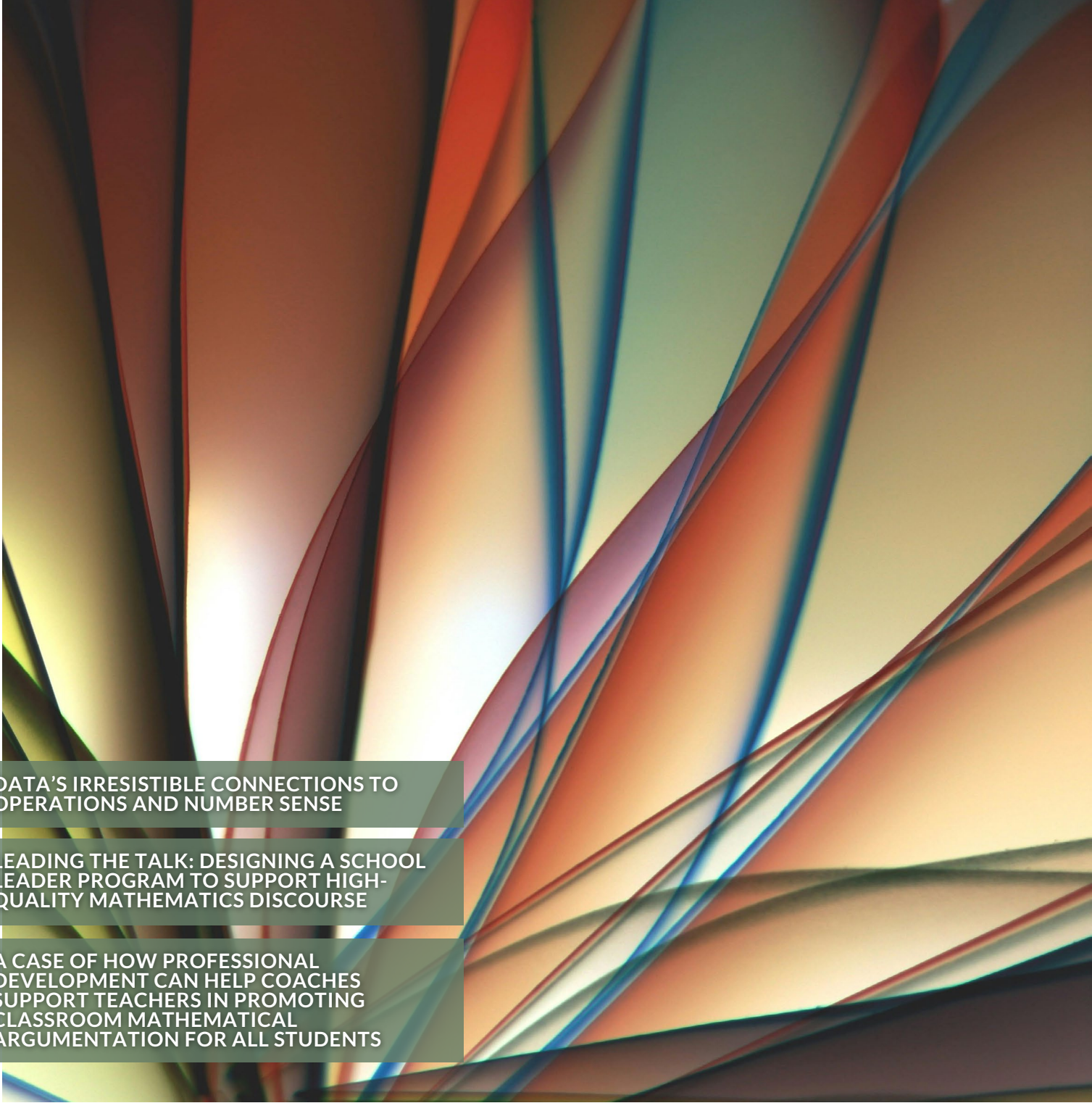


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OF MATHEMATICS EDUCATION LEADERSHIP



DATA'S IRRESISTIBLE CONNECTIONS TO
OPERATIONS AND NUMBER SENSE

LEADING THE TALK: DESIGNING A SCHOOL
LEADER PROGRAM TO SUPPORT HIGH-
QUALITY MATHEMATICS DISCOURSE

A CASE OF HOW PROFESSIONAL
DEVELOPMENT CAN HELP COACHES
SUPPORT TEACHERS IN PROMOTING
CLASSROOM MATHEMATICAL
ARGUMENTATION FOR ALL STUDENTS

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The editors of the *NCSM Journal of Mathematics Education Leadership (JMEL)* are interested in manuscripts addressing issues of leadership in mathematics education which are aligned with the NCSM Vision.

The editors are particularly interested in manuscripts that bridge research to practice in mathematics education leadership. Manuscripts should be relevant to our members' roles as leaders in mathematics education, and implications of the manuscript for leaders in mathematics education should be significant. At least one author of the manuscript must be a current member of NCSM upon acceptance for publication.

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Empirical case studies and lessons learned from mathematics education leadership in schools, districts, states, regions, or provinces;

Empirical research reports with implications for mathematics education leaders;

Professional development efforts including how these efforts are situated in the larger context of professional development and implications for leadership practice; and

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This article presents the design and evaluation of the Project All Included in Mathematics (Project AIM) School Leader Program (SLP), which supports principals in fostering high-quality mathematics discourse. We used design research to conduct three studies examining (a) SLP's design principles, (b) activity alignment with the National Council of Supervisors of Mathematics (NCSM) essential actions (EA) framework for leadership in mathematics education, and (c) principals' perspectives on SLP's contribution to their learning and leadership practices. Findings showed strong alignment between principals' needs and the design principles, consistent connections between SLP activities and EA, and key principals' perspectives, highlighting growth in instructional vision, capacity, and instructional support. Results also underscore the importance of aligning leader and teacher learning, embedding practical leadership tools, and approaching EAs as a guide when designing professional development (PD) for school leaders.

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by Anushree Bopardikar, Jennifer Knudsen, Christina B. Silva, Harriette S. Stevens

This article overviews how professional development (PD) can help coaches support teachers in fostering classroom mathematical argumentation (CMA) for all students. After summarizing conceptual underpinnings of a PD program on coaching, the article describes the program, presenting a case study of how structures and tools designed by the PD for lesson planning and reflection informed a coach's discourse with a teacher. Analyses of transcripts showed the extent to which a coach-teacher dyad discussed various topics related to mathematical concepts and practices and students' participation during early and late coaching cycles, informed by the purposes and protocols designed to visualize, plan, and debrief lesson enactment. The article highlights recommendations for PD design and research to support mathematics coaches.

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About NCSM

NCSM Mission Statement

NCSM is a mathematics education leadership organization that equips and empowers a diverse education community to engage in leadership that supports, sustains, and inspires high quality mathematics teaching and learning every day for each and every learner.

NCSM Vision Statement

NCSM is the premier mathematics education leadership organization. Our bold leadership in the mathematics education community develops vision, ensures support, and guarantees that all students engage in equitable, high-quality mathematical experiences that lead to powerful, flexible uses of mathematical understanding to affect their lives and to improve the world.

High-quality leadership is vital to this vision. NCSM is committed to:

Developing and Informing Vision

- Provide leadership to influence issues and policies affecting mathematics education in ways consistent with the mission and vision of NCSM;
- Equip leaders to be critical consumers of educational information, research, and policy to become change agents in their communities;
- Support leaders to develop an actionable vision of mathematics instruction consistent with a view of mathematics as a sense-making endeavor.

Ensuring Support to All Stakeholders

- Develop networking and communication opportunities that connect the mathematics education community as well as the broader education community;
- Equip leaders with the tools to create and sustain systems that fully align with the vision of mathematics and mathematics instruction promoted by NCSM;
- Equip leaders with the understanding, knowledge, and skills to continue their own personal growth, support emerging leaders, and further develop excellence in mathematics teaching.

Guaranteeing All Students Engage in Equitable, High-Quality Mathematical Experiences

- Provide advocacy and support regarding issues and policies affecting mathematics education in ways consistent with the mission and vision of NCSM;
- Provide resources for the implementation of research-informed instruction to ensure students engage in relevant and meaningful learning experiences that promote mathematics as a sense-making endeavor;
- Advocate for each and every student to have access to rigorous mathematics that develops their understanding, skills, and knowledge, along with the confidence to leverage their learning, in order to improve their world.

COMMENTS FROM THE EDITORS

Heather Crawford-Ferre
Nevada Department of Education
Chadd McGlone
Mathkind Global

At the beginning of a school year, attention naturally turns to planning. Teachers prepare lessons, and leaders schedule professional learning. The opening weeks also reveal who is in the room. A teacher begins to notice which students risk sharing an unfinished idea and which need more time before speaking. A coach hears how a teacher describes the classroom and what that teacher hopes to strengthen. Knowing who we lead is part of mathematics leadership because what we learn about people should affect the decisions we make with and for them. Early impressions remain provisional. A student who appears hesitant may be reasoning carefully. A teacher who seems resistant may be responding to an initiative that overlooks the realities of the classroom. Effective leaders continue examining what they think they understand, allowing new evidence to refine the support they provide.

The three articles in this issue of the *Journal of Mathematics Education Leadership* illuminate this responsibility from different vantage points. Together, they show how responsive leadership depends on learning about the people a program or partnership is intended to support.

In “Data’s Irresistible Connections to Operations and Number Sense,” McKinney and Bertolone-Smith begin with elementary teachers’ descriptions of their experiences teaching data. Many reported limited preparation and uncertainty about how data concepts belonged within grade-level mathematics. Some viewed data primarily as graphing, while others gave the topic little instructional attention. The authors used these responses to shape Data ICONS, a workshop that connects data with operations and number sense through familiar elementary routines and hands-on mathematical activity. Teachers encountered data through counting and comparison, using discussion to make those relationships visible. This framing located data within mathematics they already taught throughout the year. The workshop grew from what participants had experienced while opening a broader view of the mathematics they teach. McKinney and Bertolone-Smith show that professional learning becomes more useful when its design reflects participants’ preparation and the questions they carry into the experience. Mathematics leaders cannot assume that standards or prior professional learning have created a shared understanding. Knowing who we lead includes learning how teachers currently understand the content they are expected to teach.

In “Leading the Talk: Designing a School Leader Program to Support High-Quality Mathematics Discourse,” Yilmaz, Sztajn, Heck, Malzahn, Schnegg, Sachs, Gummer, and Riddick turn attention to principals. Their Project AIM School Leader Program helps school leaders develop the

knowledge needed to support high-quality mathematics discourse. Its design draws from principals’ expressed needs. The program aligns principal learning with the professional development teachers receive through Project AIM. It also provides resources that principals can use during classroom observations and conversations with teachers. Participating principals described a clearer instructional vision and more precise ways to support professional growth. Shared language helped them recognize the practices they hoped to strengthen across a school.

This article broadens the idea of knowing who we lead by asking those who design leadership development to understand the leaders themselves. Principals are often responsible for supporting instruction across every subject, even when their own preparation has offered limited opportunities to study mathematics teaching. Their professional learning must therefore reflect both the instructional vision they are expected to advance and the conditions under which they lead.

In “A Case of How Professional Development Can Help Coaches Support Teachers in Promoting Classroom Mathematical Argumentation for All Students,” Bopardikar, Knudsen, Silva, and Stevens examine how professional development structures inform the discourse between a coach and a teacher. The Visualize Teaching Professional Development program asked the pair to complete the mathematics together before teaching. They later used lesson video and a discussion guide to revisit what occurred. The authors trace how these routines directed attention to students’ mathematical thinking and their participation in classroom discourse. Across coaching cycles, the coach grounded her questions in evidence from the teacher’s classroom. The conversation could move beyond general impressions toward specific moments of student argumentation and participation.

The coach’s understanding of the teacher and students developed through close study of actual lessons. Knowing who we lead extends far beyond an introductory conversation. It deepens when leaders return to evidence from practice and allow that evidence to influence the next decision. The structures described by Bopardikar and colleagues embed this kind of attention within the coaching process.

Across the three articles, what leaders learn about participants alters the support they provide. That influence is visible in the design of Data ICONS and Project AIM, as well as in the classroom evidence brought into the Visualize Teaching conversations. Leadership becomes more

Continued on next page...

responsive because design begins with closer attention to the people involved.

This attention requires more than familiarity. A leader may know a teacher well and still rely on assumptions that no longer fit. Responsive leadership depends on remaining open to what changes over time and to what becomes visible through sustained interaction. The beginning of a school year offers an important opportunity to establish that habit. The knowledge developed now will influence decisions about professional learning and instructional support throughout the months ahead.

As you engage with the articles in this issue, we invite you to consider how well you know the people you lead and how that knowledge is shaping your practice. What are teachers telling you about the support they need, and what are students showing about how they experience mathematics? The answers should affect what comes next.

We are grateful for your continued commitment to mathematics education leadership and hope this issue offers ideas that support your work throughout the academic year.

DATA'S IRRESISTIBLE CONNECTIONS TO OPERATIONS AND NUMBER SENSE

Kat McKinney
California State University
Claudia Bertolone-Smith
California State University

ABSTRACT

This article discusses a workshop designed to enhance elementary teachers' understanding of how data, number sense, and operations are interconnected. Despite the important role of data in the Common Core State Standards, many teachers have reported feeling unprepared to teach data concepts. Accordingly, the workshop engaged teachers in hands-on activities (i.e., data blocks, datafied counting collections, graph-talk routines) designed to deepen their knowledge and confidence in teaching data. These activities helped teachers explore connections between data and other math concepts (e.g., comparison, counting, estimation). The article highlights the importance of viewing mathematics as an interconnected whole and offers strategies to integrate data-rich activities into classroom instruction, promoting deeper mathematical understanding and student engagement.

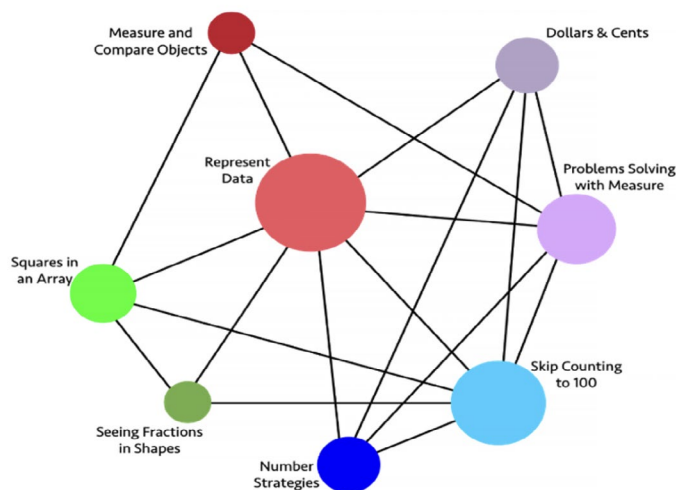
In Spring 2024, we—two scholar-practitioners—asked 44 elementary-grades teachers in northern California to respond anonymously to a survey about their teaching of data concepts. The following responses capture some of their sentiments: “I have not used data concepts outside of graphing,” “I feel that I have little understanding on what or how to teach data concepts at my grade level,” and “I don’t think I’ve thought about teaching data at all [this year].” The topic of data has served as a strand (paired with the topic of measurement) in all grade bands of the Common Core State Standards (National Governors Association & Council of Chief State School Officers, 2010) since the standards’ rollout in classrooms across the United States. Many elementary-grades educators, though, have viewed the topic of data as a tangential “if we have time” or add-on topic rather than an essential strand that holds rich opportunities for learning mathematics. We saw this sentiment expressed repeatedly in the survey results.

We also found that participating teachers expressed limited knowledge about elementary grades data topics and/or how to teach them. Indeed, this finding reflected a larger trend: Many practicing teachers nationwide do not have adequate content preparation in data and statistics (Franklin et al., 2014). Accordingly, the American Statistical Association’s *Statistical Education of Teachers* report argued for high-quality professional learning opportunities designed to help

in-service teachers advance their understanding of data and statistics (Franklin et al., 2014).

Fortunately, the notion that data topics are central to children’s mathematics education in all grades has gained traction in recent years. Notably, the California mathematics framework (California Department of Education, 2023) included at least one data-centric “big idea” in each grade. The framework highlights the link between data-focused big ideas and other big ideas via interconnected, grade-by-grade graphics, with the most connected ideas appearing larger and more centrally on the graphic. Figure 1 shows an example of an interconnected graphic from the California mathematics framework showing the big ideas of second-grade mathematics. At the center of the graphic, with the largest node, is the big idea to “represent data,” which directly connects to all seven of the second-grade big ideas (e.g., number strategies, measure and compare objects), indicating its conceptual link to each of those big ideas. Further emphasizing the importance of data topics, the framework lists “reasoning with data” as 1 of 4 cross-cutting “content connections” that span the mathematics curriculum from transitional kindergarten (TK) through Grade 12.

Figure 1
Connections Between the Big Ideas of Second-Grade Mathematics



Note. Adapted from *Mathematics Framework for California Public Schools* (p. 47), by California Department of Education, 2023 (<https://www.cde.ca.gov/ci/ma/cf/documents/mathframeworkch6.pdf>). Copyright 2023 by California Department of Education.

Beyond its connectedness to other topics, teaching mathematics through a data-focused lens can promote more equity and access for children. Indeed, data-centered activities often occur in contexts that include firsthand experiences for children and/or tap into their own funds of knowledge (González et al., 2001). Further, collecting and analyzing data can help students learn more about themselves and their worlds (Bargagliotti et al., 2020).

BACKGROUND

With these ideas in mind, we developed and gathered activities for a workshop we called Data ICONS (Data's Irresistible Connections to Operations and Number Sense). The 4-day workshop, which was cofacilitated twice by our research team, occurred in consecutive weeks of July 2024. Nineteen TK–5 teachers attended the first workshop, and 25 attended the second workshop. Teachers were from a range of counties, districts, and schools across northern California, with years of experience teaching ranging from 1 to more than 30.

Our primary goals for the workshop were to help the participating teachers (a) deepen understanding of the myriad connections between the topics of data, operations, and number sense through firsthand experiences as mathematics learners and (b) increase their confidence and knowledge of ways to incorporate data-centered activities in their own classrooms throughout the year. In this article, we share some of the key activities we used in Data ICONS for our teacher participants. We present these activities as part of three different collections representing three approaches for activity design that teachers can use to highlight connections between data, number sense, and operations. The collections, detailed in the following sections, are (a) data blocks, (b) datafied counting collections, and (c) graph-talk routines.

Data Blocks

Our first collection of activities was called data blocks. A data blocks activity entailed the use of common physical math manipulatives to represent individual numerical data points. For example, a data point with the value 85 could be represented by using place value blocks in the usual way (i.e., eight long sticks and five small cubes). In our workshop, we first used data blocks to represent each participant's data point generated in response to the question, "How old is the oldest person you've known?" (Bush et al., 2014). If the oldest person a participant had ever known was 105 years old, for example, then they used one flat hundred block and five small one cubes to represent the value of their data point. After everyone had a place value block representation of their data point, we proceeded with various data blocks activities designed to incorporate connections to number sense and operations.

One of our first data blocks activities connected data to number sense by way of comparing numerical values visually and conceptually. We asked teachers to put their place value blocks onto a small cookie sheet and meander around the room holding their trays. As participants encountered each other, they compared the values on their trays to the values

on other participants' trays without any participants saying the values aloud. By using visual comparisons of two values, participants either (a) bumped trays if the values happened to be equal or (b) briefly put the tray with the greater value above the tray with the lesser value. Figure 2 shows a pair of participants comparing their "oldest person" values in this way.

Figure 2

Two Participants Comparing the Values of Their Data Points



Note. In this example of two participants comparing the values of their data points (99 and 93), they briefly placed the tray of 99 above the tray of 93 to signify that its value is larger.

We then expanded on this value-comparing activity by having the whole group get in order from least to greatest value, again without saying their individual data value aloud to facilitate the visual and conceptual comparison of the blocks. When all participants were in a row from least to greatest data values, we considered various questions as a group to deepen our understanding of the data set. For example, we explored the following:

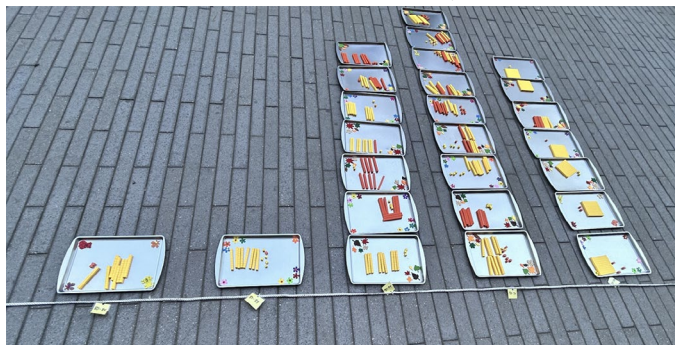
- Identifying our lowest data point and our greatest data point.
- Figuring out the difference between those values and how it related to the question, "How old is the oldest person you've known?"
- Stepping forward if a person and their neighbor (or multiple neighbors) had the exact same value. What did that mean?
- Counting simultaneously from both ends of the row to find the middle data point or points.

Next, we went outside, and participants placed their data block trays onto a number line made of rope. The rope was segmented into labeled ranges (e.g., 70–79, 80–89) to create a large bar chart representation of the data (see Figure 3). We considered new questions and ideas using this bar chart that connected our data to number sense and operations. We explored the following questions:

- Which age range has the largest number of data points, and how many data points are in that category? (Is this surprising? Why or why not?)
- Which age range has the least number of data points, and how many data points are in that category? (Is this surprising? Why or why not?)
- How could we use this chart to determine the total number of data points in our data set? (Let's do it! What strategy did you use to find the answer? What operation(s) did you use? How could we represent that strategy with an equation?)
- What would happen if our age ranges had different age spreads in them? For example, what if one category went from ages 70–74, and then the next category went from 75–79? Would this make sense to do, and why or why not?
- What would be the next age range up from our current greatest age range? How do you know? What would be the next age range down from our current lowest age range? How do you know?

Figure 3

Bar Chart Created From Data Blocks Trays



Note. This bar chart was created from a labeled rope and participants' data blocks trays.

Data blocks can be used to represent other types of numerical data, not just whole-number data points with multiple 10s. For example, fraction data points can be represented using fraction circles, pattern blocks, or fraction strips; whole-number data points with values less than 20 can be represented using snap cubes or physical ten frames with counters; and decimal data points can be represented with place value blocks by using a larger sized block as 1.

In our workshop, we used snap cubes to represent data points, all of which had whole-number values less than 10. To generate these data, we asked the statistical question, "How many peas are in a snap pea pod?" Each participant received a snap pea pod, opened it, and counted the number of peas in their pod. Then, each participant created a snap cube tower representing their number of peas. We then went through activities that mirrored those described previously for the oldest person data, with each teacher holding their

snap cube tower for the activities that called to move around the room (i.e., partner comparison; lining up in order from least to greatest; and analyzing data when in order least to greatest, including finding range, median, and mode).

Datified Counting Collections

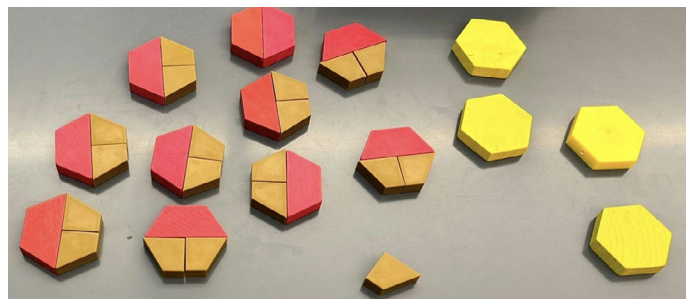
Counting collections (Franke et al., 2018) are a powerful and well-known math activity that can help students in any grade strengthen their number sense. The core of any counting collection activity is as follows: students, typically working in pairs, choose (or are given) a bag or box of items to be counted. The number of items in the collection might be small or large, depending on the student's grade level and proficiency with counting. With their partner, students empty the bags to display their collection and strategize on ways to count the total number of items. Tools to support counting are typically made available to students (e.g., empty cups or bowls, ten frames, hundred charts). After completing their count, students typically record their counting strategy on a recording sheet, making use of drawings, mathematical symbols, words, or a combination of these tools.

For our workshop, we developed activities that "datified" the standard counting collection routine. In one activity, we gave pairs of participants bags consisting of different numbers of pattern block shapes. Their task was to count the number of each different shape in their bag. Each pair counted their collections and recorded their counting strategies on individual sheets. From there, each team created a graphical representation of their counts to represent their findings; for example, some participants created a bar chart to show the number of each shape in their collection. Participants used their graphs to explore several questions connected to the concepts of number sense and operations, including the following:

- Which shape did our collection have the most of? Which shape did we have the least of? What's the difference between the amounts of these two shapes? How can we find that difference on our graph?
- Did we have the same amount of any two of our shapes? How can we see that on the graph?
- Was there any shape that we had more than 10 of? How can we see that on the graph?
- If we combined the amounts of two of our shapes, would it be more than, less than, or equal to 20? How can we use the graph to show this?

In a subsequent datified counting collections activity, we again gave each pair of teachers a bag of pattern blocks (with varying amounts of blocks in each bag). Then, in the style of a standard counting collection activity, we asked each pair to find the total "value" of their bag if the area of a yellow hexagon was worth 1. Figure 4 presents an example of one team's counting strategy.

Figure 4
Example Team Strategy for Finding Total Value



Note. This photo depicts one team's strategy for finding the total value of their collection (i.e., $14 \frac{1}{4}$) if the area of a yellow hexagon is worth 1.

After each participant recorded their counting strategy on their own paper, each team wrote the total value of their bag on the whiteboard at the front of the class, which then formed a data set. Together, using our class data set, we coconstructed a line plot with fractional values and proceeded to have a number talk (Parrish, 2010) about the difference between the bag with the largest value and the bag with the smallest value.

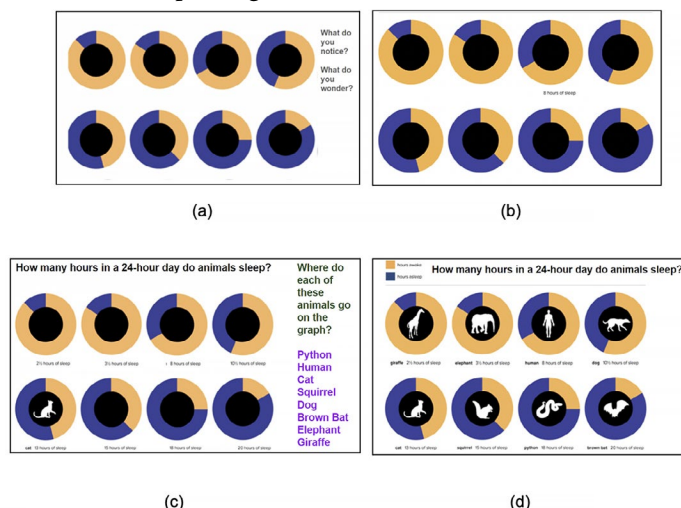
Graph-Talk Routines

In our workshop, we used two graph-talk routines—slow reveal graphs and data talks—to provide our participants with further opportunities to engage in nuanced discussion and reasoning about data represented in graphical form. The slow reveal graph (Laib, n.d.) routine begins with an entirely unlabeled graph on the first slide in a projected slideshow, and the facilitator advances through the slideshow to reveal more labels on the graph incrementally. Participants are engaged via reflective questions and discussion with every new reveal. As a more concise, graph-based routine, data talks (Boaler et al., 2021) involve the analysis and discussion of a complete graph projected onto a screen. In this routine, all details of the graph are revealed from the beginning, which allows for participants to notice, wonder, and discuss any features of the graph that catch their attention.

Slow Reveal Graphs

We used slow reveal graphs (Laib, n.d.) to generate intense focus on the structure, values, context, and story of graphs and data representations, helping participants pay particular attention to the connections to numbers and operations. The first step in a slow reveal is showing the graph without any context or number clues. An example created from a graph in the book, *Animals by the Numbers: A Book of Infographics* (Jenkins, 2016) and found on the website, “Slow Reveal Graphs” (Laib, n.d.), appears in Figure 5(a). Participants examined the structure of this “bare” graph and reflected on what they noticed and wondered.

Figure 5
Slow Reveal Graph Progression

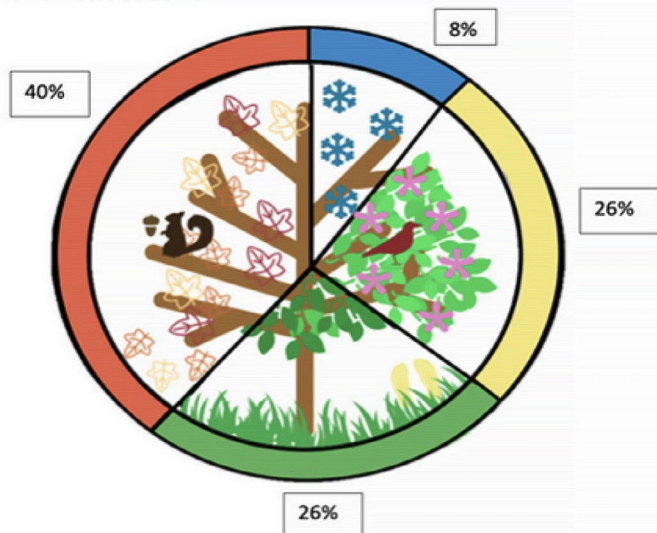


Note. These images depict a slow reveal graph progression (Laib, n.d.).

As a slow reveal graph routine continues, information is revealed bit by bit (see Figure 5b–5d). Each time new information was revealed, we asked participants to consider what they knew about the graph and its underlying data. In the example from our workshop that appears in Figure 5, it was first revealed that one circle was labeled “8 hours of sleep” (see Figure 5b). Then, we challenged participants to label all circle values using estimation and the proportionality of the graph. Next, the context of the graph was revealed via the graph’s title, “How Many Hours in a 24-Hour Day Do Animals Sleep?” The number of hours a squirrel sleeps was also revealed, along with the list of the other animals on the graph (see Figure 5c). Participants were then asked to guess where each animal might go on the graph. In doing so, participants shared their funds of knowledge about different animals and their characteristics that may indicate sleeping patterns. For the grand finale, we shared the entire graph (see Figure 5d). A chorus of “Oh!” and “I knew it!” with smiles and high-fives erupted around the room.

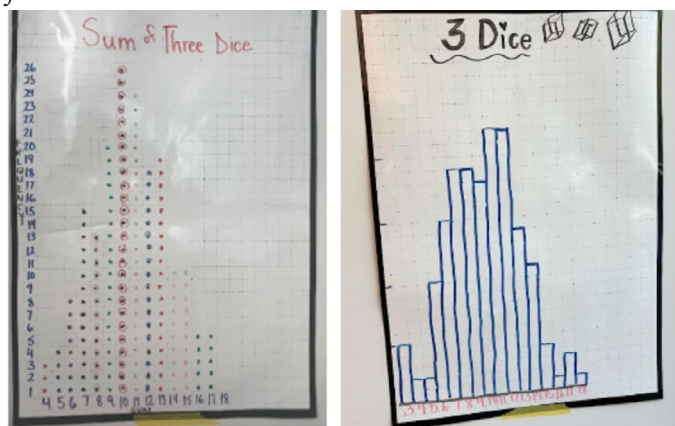
Data Talks

We also introduced the instructional routine of data talks (Boaler et al., 2021). Data talks begin with a graph of all information present from the outset (see Figure 6) and the questions, “What do you notice? What do you wonder?” During the workshop, participants compared data provided on the graph to their favorite seasons and wondered why so few people liked winter. A strong numerical comparison of data and connection to the graph context often emerged during our data talks. As the week progressed, we noticed a more sophisticated understanding.

Figure 6*Four Seasons Data Talks Example***Favorite Seasons**

Note. Values were included to support number sense and scale within the graph, provoking further data thoughts (Boaler, n.d.).

Initially, we used premade data representations for our data talks; however, many of our Data ICONS activities generated data and graphs of their own. For example, we collected data on the sum of three dice. Teams created a variety of data representations of their results, and we then used these graphs for data talks (see Figure 7).

Figure 7*Sums of Three Dice Graphs Created by Participants and Used for Data Talks*

Our data talks about participant-generated graphs led to deeper discussions on underlying reasons for the data collection results. For example, participants discussed which representation was more effective (i.e., dots or bars), the similarities and differences of the collected data, and why 11 and 12 were the most popular sums when rolling three dice.

CONCLUSION

Connections are at the heart of robust mathematical understanding. In her book, *Limitless Mind*, Boaler (2019) underscored the dire importance of every mathematics learner understanding mathematics as an interconnected subject rather than a series of isolated concepts to memorize. Boaler warned, without this concept of interconnectedness, teachers may inadvertently stifle students' ability to see the beauty and utility of mathematics in their lives. Moreover, Ma (1999) famously argued that a deep understanding of the interconnectedness of mathematical concepts is a vital part of teachers' knowledge. Only when teachers see the links between mathematical concepts can they plan math activities that position their students to discover those connections for themselves. Teachers must build a coherent and connected understanding of mathematics for their students rather than a fragmented one.

As we illustrated with the collections of activities described in this article (i.e., data blocks, datafied counting collections, and graph-talk routines), the topic of data yields a perfect foundation for discovering and experiencing the interconnectedness of mathematics. With data blocks, the values of data points can be compared, ordered, and grouped visually and conceptually. With datafied counting collections, collections and their counts become data for a graph or a number talk. With graph-talk routines, estimation and interpreting numbers in context become focal points for discussion and deep reasoning. With these activities (and others), teachers and their students alike can enjoy data-filled experiences throughout the academic year while deepening their understanding of all strands of the mathematics curricula.

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LEADING THE TALK: DESIGNING A SCHOOL LEADER PROGRAM TO SUPPORT HIGH-QUALITY MATHEMATICS DISCOURSE

ABSTRACT

This article presents the design and evaluation of the Project All Included in Mathematics (Project AIM) School Leader Program (SLP), which supports principals in fostering high-quality mathematics discourse. We used design research to conduct three studies examining (a) SLP's design principles, (b) activity alignment with the National Council of Supervisors of Mathematics (NCSM) essential actions (EA) framework for leadership in mathematics education, and (c) principals' perspectives on SLP's contribution to their learning and leadership practices. Findings showed strong alignment between principals' needs and the design principles, consistent connections between SLP activities and EA, and key principals' perspectives, highlighting growth in instructional vision, capacity, and instructional support. Results also underscore the importance of aligning leader and teacher learning, embedding practical leadership tools, and approaching EAs as a guide when designing professional development (PD) for school leaders.

Research has highlighted the important role principals play in the success of their schools, teachers, and students. Principals' support for teachers' professional learning enables teachers to adapt continually to the complexities of teaching mathematics (Nelson, 1998). Strong principal support has been linked to improved teacher attitudes, preparedness for instruction, and implementation of effective classroom practices (Heck et al., 2008). Principals account for approximately one quarter of school effects (i.e., second only to teachers) on student outcomes (Leithwood et al., 2004), making principals and their work a key phenomenon of interest for student success.

Although studies on the importance of principals as instructional leaders have gained attention in the education leadership literature, relatively few scholars have focused on how to support principals as they develop the knowledge and practices needed to promote teachers' professional growth and instructional improvement. More

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specifically, in mathematics, despite the well-documented importance of principal practices for promoting high-quality mathematics instruction (HQMI), professional development (PD) efforts for principals in the context of mathematics reform initiatives have often focused narrowly on developing principals' content knowledge (Boston et al., 2017; Steele et al., 2015) or on classroom management and organizational strategies (Cunningham & Lochmiller, 2020). The lack of coherent systems connecting leadership roles (e.g., principals) with teacher learning poses a significant barrier to sustained instructional improvement in mathematics (Kazemi & Resnick, 2020). Therefore, it is important to design PD that build principals' capacity to foster instructional practices that support HQMI, aligning leadership development with teacher learning.

In this paper, our research team shares a set of design principles and activities used in a PD program for school leaders interested in fostering HQMI. We contend these tested principles and types of activities can support other school leaders interested in augmenting their knowledge of how to support quality instruction for teachers and learners at their own schools. As part of a larger design research study, we developed the School Leader Program (SLP) as one component of the larger initiative called Project All Included in Mathematics (Project AIM). In what follows, we first briefly review the literature on principals' knowledge for promoting HQMI. Next, we provide information about Project AIM, the context for the AIM-SLP, and its focus on high-quality mathematics discourse (i.e., a key feature of HQMI). Then, we share the design principles and related activities created for the AIM-SLP. Finally, we present the results of three short studies used to test our design. These studies examined the following questions:

- In what ways do the PD principles derived from the literature align with what school leaders consider the knowledge, skills, and resources needed to implement high-quality mathematics discourse?
- In what ways do the designed AIM-SLP activities align with the essential actions (EA) framework for leadership in mathematics education outlined by the National Council of Supervisors of Mathematics (NCSM)?
- What are the perspectives of participating principals regarding the contributions of the AIM-SLP to their support for high-quality mathematics discourse?

Principals' Knowledge and Practices for Promoting HQMI

In mathematics, principals play a crucial role as school instructional leaders by establishing instructional vision, ensuring teachers have necessary resources and professional

learning opportunities, and creating conditions that enable teachers to deliver HQMI (Boston et al., 2017; Cunningham & Lochmiller, 2020). Supporting HQMI necessitates that principals cultivate a shared vision of teaching and learning rooted in meaningful and engaging mathematics learning experiences for all students (Cobb et al., 2018; Munter, 2014). Effective leadership practices that support instructional improvement in mathematics include fostering teacher collaboration through professional learning communities, conducting classroom observations, providing constructive feedback, and building professional capacity (Goddard et al., 2015; Hitt & Tucker, 2016; Kazemi & Resnick, 2020). Principals who prioritize these leadership activities tend to cultivate positive school cultures that promote academic success for all students (Sebastian et al., 2018; The Wallace Foundation, 2013).

Supporting HQMI also involves developing a shared vision of teaching and learning rooted in meaningful and engaging mathematics learning experiences for all students (Cobb et al., 2018; Munter, 2014). Research has suggested principals need mathematical and pedagogical content knowledge to engage meaningfully in instructional discussions with teachers and support instructional practices that promote student learning and engagement (Corcoran, 2017; Cunningham & Lochmiller, 2020). This expectation presents a challenge for principals who must support high-quality instruction in all content areas. In fact, some researchers have considered whether principals can realistically become content specialists for every subject area (Fink & Resnick, 2001).

Stein and Nelson (2003) addressed this tension through the concept of leadership content knowledge, which proposes that “principals need an enhanced understanding of effective teaching and learning practices in specific content areas” (Boston et al., 2017, p. 184) rather than deep subject expertise. Boston et al. (2017) also stated, “Principals need sufficient knowledge of what constitutes HQMI” (p. 184), which can help them effectively conduct classroom observations, provide helpful feedback to teachers, and establish a clear vision with high expectations for mathematics instruction (Kazemi & Resnick, 2020).

One effective strategy for principals to enhance their understanding of HQMI and become attuned to teachers’ needs is to engage as colearners in teacher PD focused on mathematics content and pedagogy. Through active participation in teacher PD sessions, team meetings, and collaborative lesson planning, principals can deepen their understanding and knowledge of effective mathematics instruction and reinforce their roles as instructional leaders (Cunningham & Lochmiller, 2020; Goddard et al., 2015; Grissom et al., 2021; Kazemi et al., 2022). This engagement not only strengthens their capacity to support instructional improvement, but also fosters continuous learning and collaboration within school culture (West, 2017). For many principals, time constraints and competing demands require strategic choices about when to participate in PD designed for their teachers.

Although sufficient knowledge is crucial for instructional leadership, effective principals are also expected to apply this knowledge in specific leadership practices to support HQMI. Several research-documented practices (e.g. Boston et al., 2017; Hitt & Tucker, 2016) can strengthen support for effective instruction. First, beyond establishing a shared instructional vision, principals must align school goals and communicate expectations that reflect a commitment to this vision (Hitt & Tucker, 2016). Integrating this vision with classroom practices requires that principals communicate expectations for HQMI (Boston et al., 2017; Hitt & Tucker, 2016) and enhance their own understanding of HQMI while attending to teachers’ professional learning needs (Kazemi & Resnick, 2020; Nelson, 1998). This dual focus on their own growth and that of their teachers positions principals as collaborative leaders who can connect instructional vision and classroom implementation (Nelson, 1998).

Principals who provide consistent instructional support to teachers and create professional learning opportunities are better positioned for effective implementation of HQMI. This support can include leadership activities like reflecting on lesson plans, conducting classroom observations, providing actionable constructive feedback, and creating reflective spaces for professional growth (Grissom et al., 2021; Hitt & Tucker, 2016; Sebastian et al., 2016). Principals play an important role in addressing contextual challenge to foster a supportive school environment (e.g., limited time, resources, and opportunities for teacher learning; LaVenia, 2015; May & Supovitz, 2011). In addressing these challenges, principals can distribute leadership by working collaboratively with teacher leaders and instructional coaches to build instructional support capacity across the school (Cunningham & Lochmiller, 2020; Hitt & Tucker, 2016). This collaborative leadership approach ensures coaches, teachers, and principals support teachers’ learning collectively and attend to a shared vision and goals for instruction.

Trustful relationships and accountability are fundamental elements to the success of key leadership practices. Trust between principals and teachers fosters open communication, collaboration, and a shared commitment to HQMI and instructional improvement. Principals build trust by providing dependable support, recognizing teachers’ needs, and valuing teachers’ input in decision-making processes (Hitt & Tucker, 2016). A culture of trust encourages teachers to implement newly learned instructional practices, seek feedback, and engage in continuous professional growth, all of which contribute to enhanced student learning outcomes (Hitt & Tucker, 2016). Accountability includes setting measurable goals, monitoring progress, and providing constructive feedback to staff. By growing a culture of responsibility, principals encourage teachers to take ownership of their professional growth and student outcomes (Hitt & Tucker, 2016).

The research on effective instructional leadership practices of (a) establishing a shared vision, (b) providing consistent instructional support, (c) addressing contextual challenges, (d) distributing leadership, (e) cultivating trust, and (f) ensuring accountability informed the creation of design

principles for our AIM-SLP. These practices also aligned with established frameworks in mathematics education leadership such as the NCSM's (2020) EA framework for leadership in mathematics education, which provides structured guidance for translating research-based practices into actionable principles for leaders at all levels. In what follows, we return to the EA framework as a tool to examine AIM-SLP.

CONTEXT: PROJECT AIM

Project AIM is a long-term PD initiative designed to enhance K–2 mathematics instruction. Project AIM focuses on the promotion of high-quality mathematics discourse in the classroom, a key feature of HQMI. In Project AIM, *mathematics discourse* is defined as “patterned ways of using questioning, explaining, listening, and different modes of communication in the classroom to promote conceptual understanding in math for all learners” (Sztajn et al., 2021, p. 7). High-quality mathematics discourse uses different patterns to support all strands of mathematics proficiency (Kilpatrick et al., 2001). This type of discourse “is purposeful and engages students in taking responsibility for their own learning and for the learning of their peers.” (Sztajn et al., 2021, p. 12).

Beginning in 2010, Project AIM received a series of grants from the National Science Foundation to develop and test comprehensive PD materials for K–2 mathematics. These materials, now stored and facilitated through a learning management system, were created for scalable, district-led implementation and include a core teacher PD program (AIM-PD) and facilitator preparation program (AIM-FP). Classroom-, school- or district-level mathematics leaders who participate in the AIM-FP learn about the AIM-PD and receive access to all AIM-PD materials. They can then locally facilitate the 30-hour AIM-PD for teachers, which is designed for synchronous participation of school teams.

Our research team tested and revised the AIM-PD materials across six implementation cycles that involved over 230 teachers and 14 facilitators. Findings from these implementations demonstrated improvement in teachers' mathematical knowledge for teaching, beliefs about importance of student discussion, preparedness to facilitate discourse-rich instruction, frequency of planning for discourse, and use of student discourse methods during instruction (Sztajn, 2021). In Project AIM-PD, teachers engage with a variety of mathematics discourse techniques (Sztajn et al., 2021). The AIM-PD “flipped the script” (Guskey, 2020) on PD design to initiate new teaching practices rapidly. AIM-PD starts with lesson organization, moves quickly into the implementation of purposeful AIM discourse techniques, and circles back to connect teachers' reflections on implementation to their knowledge about high-quality mathematics discourse (Sztajn et al., 2020). Throughout the PD, teachers apply the techniques they learn in their classrooms, use their experiences to refine their implementation, and deepen their understanding of high-quality mathematics discourse and how it enhances student learning.

To further enhance the AIM-PD impact, it is important that teachers transform what they learn in the PD into action. Thus, we designed two additional programs aligned with the overall goals of Project AIM: the AIM Coaching Program (AIM-CP) for instructional leaders and the AIM-SLP for school principals, the latter of which is the focus of this paper.

THE PROJECT AIM-SLP

AIM-PD focuses primarily on classroom discourse, 1 of 3 dimensions of HQMI alongside (a) the nature of mathematical tasks and (b) the role of the teacher (Munter, 2014). The nature of mathematical tasks includes selecting high-quality tasks that challenge students to engage with various problem-solving strategies, connect their ideas to mathematical concepts, use multiple representations, and justify their thinking rather than simply focusing on procedural calculations (Munter, 2014; Stein & Smith, 1998). The role of the teacher extends beyond delivering content to involve the orchestration of classroom activities that promote mathematical exploration and position students as active thinkers and doers of mathematics. Classroom discourse involves creating a learning community where students engage in meaningful mathematical conversations, critique ideas, and justify their reasoning (National Council of Teachers of Mathematics, 2014).

The design of AIM-SLP emphasizes to principals the importance of high-quality discourse to create a learning environment where students actively explore mathematical ideas and take ownership of their learning, supported by teacher facilitation and challenging tasks (Munter, 2014). This type of mathematics discourse is purposeful and inclusive, requiring student interactions and engagement to promote conceptual understanding of mathematics and build procedural fluency and productive dispositions (Sztajn et al., 2021). It involves (but is not limited to) consistent patterns of questioning, explaining, listening, and multimodal communication embedded into daily classroom norms. Teachers play a vital role in fostering these patterns, ensuring all students participate meaningfully in classroom discourse (Sztajn et al., 2021) by listening, justifying, critiquing, and making sense of mathematical ideas (Dillon et al., 2022; National Council of Teachers of Mathematics, 2014; Stein & Smith, 1998).

Recognizing the central role of school leaders in supporting instructional improvement, AIM-SLP places a strong emphasis on principals' understanding of high-quality mathematics discourse as a key factor in driving meaningful change. This emphasis is supported by a growing body of research indicating that principals' conceptions of mathematics instruction directly influence their leadership practices and the instructional quality in their schools (Kazemi et al., 2022). Principals' visions are more predictive of teachers' understanding and enactment of instructional goals than the time principals spend engaging in instructional leadership activities directly (Katterfeld, 2013). Therefore, it is important to support principals in developing a clear understanding of high-quality mathematics discourse for

strengthening coherence between leadership practices and classroom instruction.

Using Design Research in the Development of the AIM-SLP

In Project AIM, we applied educational design research to develop our PD materials, including the AIM-SLP. Educational design research is an iterative process of systematic inquiry for creating and testing effectiveness of interventions “with the intent of producing new theories, artefacts, and practices that account for and potentially impact learning and teaching in naturalistic settings” (Barab & Squire, 2016, p. 2). The approach aims to create practical solutions to educational problems, generate transferable design principles, and contribute to existing theoretical knowledge (Plomp & Nieveen, 2013; van den Akker et al., 2006).

The educational design research process typically involves iterative cycles of analysis, design, implementation, and evaluation (Aşık & Yılmaz, 2017; Gravemeijer & Cobb, 2006). It begins with user needs and integrates user perspectives throughout (Schön, 1983) to generate tested design principles and activities. Design research also grounds PD development in existing scholarship and connects theory with practice in context (Barab & Squire, 2016; Plomp & Nieveen, 2013). It iteratively (a) tests and refines design principles and activities based on user insights and existing frameworks and (b) identifies effective elements of the PD and areas needing improvement. This process yields knowledge in the form of transferable design principles to inform the design of PD programs for learners across diverse contexts.

We engaged principals and PD experts early in the process, supporting the coconstruction of our design principles and learning activities that are responsive to school contexts, leadership needs, and mathematics learning goals. In what follows, we share the design principles and learning activities created for our AIM-SLP.

The AIM-SLP Design Principles

Based on our review of the literature, interactions with school principals, and the goals of the AIM-PD for teachers, we identified four key design principles for developing effective PD programs for principals focused on promoting high-quality mathematics discourse in the classroom:

- Draw on what is known regarding principals’ essential knowledge and practices to set PD goals that promote high-quality mathematics discourse.
- Align principal and teacher learning goals to select content that can support a schoolwide vision for high-quality mathematics discourse.
- Include principal educators in the design process to create feasible and relevant programs.
- Include practical, job-embedded tools that make the PD useful and easily applied to principals’ practice.

AIM-SLP Activities Design

AIM-SLP is a fully asynchronous, job-embedded PD initiative designed to empower school leaders to support

and enhance high-quality mathematics discourse in K–2 classrooms. In designing the AIM-SLP, the Project AIM team capitalized on our review of the literature, particularly the four design principles that emerged from this review.

Design Principle 1: Draw on What is Known

The AIM-SLP design emphasizes instructional leadership by equipping school principals with knowledge and practices necessary to foster a school culture grounded in a shared vision and goals for high-quality mathematics discourse. The program prioritizes fostering high-quality mathematics discourse and recognizes it as a key driver of student engagement, conceptual understanding, and productive learning opportunities. To operationalize this vision, AIM-SLP is structured around a set of clearly defined goals that guide school leaders in implementing effective leadership practices:

- Learn what constitutes high-quality mathematics discourse for all students in elementary classrooms.
- Understand how classroom discourse supports opportunities for all students to learn mathematics concepts, develop procedural fluency, and build productive dispositions.
- Prepare for and practice supporting teachers and coaches in their work to improve mathematics instruction that engages each and every learner in high-quality mathematics discourse.
- Develop practices to strengthen school culture for improving mathematics instruction.

Design Principle 2: Align Principal and Teacher Learning Goals

AIM-SLP content is designed to align school leader learning with foundational goals and ideas from AIM-PD for teachers, focusing specifically on high-quality mathematics discourse and instructional techniques to promote it (Sztajn et al., 2021). Modules in AIM-SLP deepen leaders’ understanding of how effective classroom discourse creates opportunities for students to develop conceptual understanding, procedural fluency, and productive dispositions. These modules also explore discourse techniques for supporting all learners, equipping principals to recognize and promote effective instructional practices. Through this alignment, AIM-SLP ensures school leaders are prepared to support teachers and coaches in implementing instructional strategies that engage all students in high-quality mathematics discourse.

AIM-SLP is comprised of eight modules progressing from foundational knowledge building to practical application. Modules 1–5 lay the foundation, building essential knowledge necessary for leading instructional improvement. These modules introduce Project AIM’s goals and structure, establish a vision for HQMI and discourse, and explore resources that support effective instructional practices for teachers. Modules 6–8 shift the focus to application, providing leaders with tools and strategies to foster teachers’ professional growth and establish a collaborative school culture centered on mathematics improvement. The overall structure of AIM-SLP is shown in Figure 1.

Figure 1
AIM-SLP Structure

Self-Guided Learning Modules					Academic Year Practice Modules		
1	2	3	4	5	6	7	8
School Leader Program Overview	What is Project AIM? What is My Role?	A Vision of High-Quality Math Discourse for All Students	Engaging Emergent Multilingual Learners in Math Discourse	Preparing for Practice	Supporting Teachers' Professional Learning	Supporting Teachers and Coaches to Improve Math Instruction	Enhancing School Culture to Promote High-Quality Math Instruction for All Students
Knowledge Building					Application to Practice		
Tools Resources for Supporting Teachers' Math Instruction							

Design Principle 3: Include Principal Educators

The AIM-SLP design recognizes the challenges (e.g., time, resources) faced by school leaders; its design features were carefully created in partnership with principal educators who understood these challenges and promoted realistic expectations for principal engagement. Thus, the self-guided nature of the program allows principals to progress through the content at their own pace and accommodates principals' demanding schedules. The eight modules of the program are designed to be completed in a total of approximately 8–12 hours during the school year, with the more engaging and time-intensive modules (i.e., Modules 1–5) intended for summer months when schedules are more open.

AIM-SLP also intentionally incorporates a variety of relevant and flexible instructional activities tied closely to principals' everyday leadership roles and responsibilities (e.g., classroom observations) and provides access to foundational content and skills needed to support teachers' implementation of HQMI and discourse practices. Sample activities include observing portions of the AIM-PD, studying videos of classroom instruction, responding to reflection prompts, and engaging with pertinent readings. These activities support principals in (a) identifying their needs, (b) assessing their own growth in knowledge, (c) setting actionable leadership goals to support HQMI and discourse, and (d) adapting what they learn to their unique school contexts. For instance,

in interactive video activities, principals enhance their professional learning on instructional leadership. The video platform pauses the videos at strategic moments of classroom instruction or teacher PD to present embedded reflection questions that prompt principals to consider how to respond at key moments or to envision how important ideas apply in their own leadership contexts.

Design Principle 4: Include Practical, Job-Embedded Tools

AIM-SLP supports principals in enacting practices directly in their school contexts. Resources such as the discourse analysis record sheet, classroom walkthrough checklist for use during observations of instruction, and conversation guides for discussions with teachers (see Figure 2 for an example of a conversation guide on reflecting on a mathematics lesson) were designed to integrate easily into leaders' daily routines. The program minimizes added workload while maximizing instructional impact by embedding these tools in existing leadership practices. These resources support principals in conducting focused observations, facilitating reflective discussions, and providing constructive feedback to drive instructional improvement. A resource library provides ongoing access to these tools so principals can use them to sustain efforts for promoting high-quality mathematics discourse after the program's completion.

Figure 2

Conversation Guide: Reflecting on a Mathematics Lesson

Project AIM PD: School Leader Program

Project AIM School Leader Resource
Conversation Guide: Questions for Reflecting on a Mathematics Lesson

After a teacher has taught a lesson aimed at promoting responsive discourse (whether you observed the lesson or not), you can use a subset of these questions as a guide to support the teacher's reflection. The lead-in questions will help you get a sense for which parts may be appropriate for the particular lesson.

Ask parts A and B to discuss the goals and task(s) of the lesson.

A. Setting Mathematics Content and Discourse Goals

Lead-in: Can you say a little bit about mathematics and discourse goals for the lesson?

B. Selecting a Task for Promoting Responsive Discourse

Lead-in: Can you describe your planning process for selecting the lesson task(s)?

1. How well did the lesson task(s) engage students with the mathematics and discourse goals?

Choose ONE of part C, D, or E to focus on a particular phase of the lesson.

C. Launching the Task <i>Lead-in: What considerations did you keep in mind when launching this task?</i> 1. In what ways did your launch prepare all students, including emergent multilingual learners, to engage with the task? What wasn't as effective or didn't prepare all students to engage? [You might follow up about: the context of the task, scaffolding questions, word walls/banks, encouraging various modes of communication]	D. Exploring the Task & Rehearsing to Share <i>Lead-in: How did you organize students for learning in this lesson so that all had opportunities to participate in responsive discourse?</i> 1. How well did the organization actively engage all students, including emergent multilingual learners, with Exploring the mathematics of the task? With engaging in responsive discourse through their [questioning, explaining, listening, and using various modes of communication]?	E. Discussing and Making Sense of the Mathematics of the Lesson <i>Lead-in: How did the lesson enable all students to further their mathematical thinking?</i> 1. How well did the Discuss support responsive discourse for all students, including emergent multilingual learners, in the ways they [questioned, explained, listened, and used various modes of communication]? 2. How well did the lesson make use of multiple solutions, including incorrect ideas, to deepen students' mathematics understanding?
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Ask part F to look ahead from this lesson to improving future instruction.

F. Looking Ahead to Future Instruction

1. What did you learn from this lesson that you want to remember for future instruction?
2. What would you like to improve for future instruction? What can the school do to support you in making those improvements?

STUDIES

To examine the relevance of AIM-SLP design principles and activities, we conducted three small, focused studies. Each study explored a distinct aspect of the program design and its implementation. The first study examined the alignment between our program design principles and what school leaders identified as essential and needed for support of high-quality mathematics discourse. The second study explored how the AIM-SLP activities reflect the recommendations expressed in the EA framework for leadership in mathematics education (NCSM, 2020). The last study examined how principals participating in the AIM-SLP perceived its impact on their instructional leadership practices. Together, these studies provided insight into how the program supports school leaders, informed revisions and refinements of the program, and offered design principles that can be shared with the broader field to guide similar professional learning efforts.

Study 1: AIM-SLP Design Principles' Alignment With Principals' Needs

Study 1 aimed to answer the first research question: In what ways do the PD principles derived from the literature align with what school leaders consider the knowledge, skills, and resources needed to implement high-quality mathematics discourse? To inform the design of AIM-SLP, the research team conducted a focus group with school leaders who were familiar with Project AIM and had teachers participate in the PD ($n = 5$). The focus group interview was aimed at understanding (a) what principals need to know about HQMI and discourse, (b) leadership practices for supporting teachers and cultivating a school culture that promotes high-quality mathematics discourse, and (c) types of tools and resources that are most helpful to principals in promoting high-quality mathematics discourse. We also inquired about the benefits and challenges of the proposed AIM-SLP PD, including its anticipated length and feasibility of a self-paced model.

The focus group data were analyzed to examine how school leaders expressed needs and insights on supporting high-quality mathematics discourse aligned with the foundational principles guiding the AIM-SLP development. Our findings are organized around each of the design principles.

Design Principle 1: Draw on What is Known

Analysis of the focus group interview data showed principals need a clear and shared knowledge and understanding of what constitutes effective mathematics instruction and productive mathematics discourse. This finding affirmed the first AIM-PD design principle of SLP. As Principal Becky explained:

I think just an underlying understanding, first of all, what effective mathematics instruction is and isn't. So, there needs to be sort of a basis for that . . . the supporting pieces for mathematics, this mathematical discourse. What does that look like? Why is it effective? Why is it important?

In addition to the knowledge of high-quality mathematics discourse, principals emphasized the need for specific

leadership skills, particularly the ability to engage teachers in reflective conversations that promote professional growth. As Principal Jack shared, "Principals need to be comfortable asking questions and learning from their teachers." Similarly, Principal Jennifer expressed, "The leadership skill is asking the right questions to get your teachers to reflect," underscoring the role of strategic inquiry in supporting teacher learning. These insights aligned with research recommendations (e.g., Grissom et al., 2021; Kazemi & Resnick, 2020; Kazemi et al., 2022) and reinforced the importance of grounding PD design in leadership knowledge, skills, and practices that directly support principals' instructional leadership for teacher reflection and growth.

Design Principle 2: Align Principal and Teacher Learning Goals

Leaders also emphasized the principals' role in developing collaborative learning spaces that ensure coherence in mathematics instruction across grade levels. They noted principals need to be aware of the focus of teacher learning to align leadership practices to improve discourse and HQMI. This insight validated the principle of aligning principal and teacher learning goals. For instance, Principal Jeremy highlighted the need for principal PD aligning with teacher PD for better supporting teachers, saying:

I think that with our first round, I felt like our first challenge was that we didn't know how to support our teachers through this professional development, because we didn't have [PD] . . . I was envisioning this [SLP] being paired alongside our teachers in Project AIM . . . simultaneously as they were attending [the AIM-PD].

Similarly, Principal Becky emphasized the need for aligning principals' learning with the teachers' learning. She said:

Learning along with your teachers, learning something that you really haven't seen, that you weren't trained in, that you probably didn't really practice in your own classrooms back in the day. So, you know, learning it to see what that looks like, and then being willing to engage with it with your teachers.

Design Principle 3: Include Principal Educators

Principals offered valuable input about the knowledge, skills, and practical tools they considered most useful and relevant in the SLP, and their input aligned directly with recommendations provided by the principal educator who was part of our team. The principals emphasized that program modules should be connected to classroom observations, manageable in length (i.e., no more than 1–1.5 hours per module, totaling under 10 hours), and include flexibility by offering foundational and optional content to meet the different stages of leadership development. Principals also suggested structuring the program to (a) be more engaging in the summer, when they have more time; (b) include a midyear virtual check-in with peers; and (c) culminate in a final spring session to reflect on progress, identify needs, and share experiences. These insights from the principals aligned with the recommendations, content,

and structure our partnering principal educator advised for the AIM-SLP.

Design Principle 4: Include Practical, Job-Embedded Tools

Principals indicated they needed to understand and have the necessary resources to conduct intentional observations in classrooms and to provide meaningful feedback to teachers. These recommendations supported our fourth principle of including practical, job-embedded tools. For instance, Principal Jennifer shared:

What should I expect to see in that mathematics classroom? Almost like, what, if our district's working on our walkthrough instrument for mathematics, what would be some things that we would put on there to make sure that we're seeing?

Principal Jennifer's reflection validated our design principle of including job-embedded, practical tools that help principals identify characteristics of HQMI and discourse as part of their classroom observation process. Similarly, Principal Jack emphasized the usefulness of resources aligned with teacher PD, sharing, "the *Activating Math Talk* book [Sztajn et al., 2021] is really helpful. We're doing some work with that as a follow-up to implementing Project AIM and [Cognitively Guided Instruction], and that's been a great resource for us to use." Principal Jeremy also suggested the program should provide some guiding questions to engage in reflective conversations with their teachers and said, "These are some ways that you could support them. Here's the critical pieces that you need to know. Here's some great questions that you could ask."

Study 2: NCSM EAs for Guiding Principles Alignment for AIM-SLP

Study 2 aimed to answer the second research question: In what ways do the designed AIM-SLP activities align with the EA framework for leadership in mathematics education outlined by the NCSM? Conducted by Project AIM's external evaluator, this study included a content analysis by aligning the AIM-SLP program's content with EAs in NCSM's (2020) framework for leadership in mathematics education to identify sources of evidence for potential alignment.

The framework records emerging themes in mathematics leadership through leadership guiding principles and indicators within each principle (see Table 1). These indicators embody "big ideas" and stages mathematics education leaders are likely to experience as they grow and evolve into their positions (NCSM, 2020), from a focus on or commitment to the leadership of the self to the leadership of others and the leadership of the extended community. The framework can be used to identify the knowledge and skills that might be targeted in PD programs for education leaders.

In the AIM-SLP alignment study, the project evaluator translated Table 1 into a spreadsheet that listed the EA for each guiding principle in rows and headed the columns with the AIM-SLP activities from the learning management system. Alignment of the contents of the activities to each EA was then scored in the resulting cells. One Project

AIM researcher also independently scored the contents of the activities in five selected modules to each EA. The discrepancies were resolved through consensus discussion. Table 2 shows a snapshot of the analysis spreadsheet.

We reviewed each AIM-SLP activity to consider its potential to provide leaders an opportunity to engage with key concepts of the EAs. The score in each cell in Table 2 represents the extent to which all instructional elements comprising the activity provided an opportunity to learn about what the EA meant in the context of Project AIM. Score definitions are:

- 0: the key concepts of the EA were not addressed in the activity
- 1: the key concepts of the EA were mentioned in the activity
- 2: the key concepts and/or skills of the EA were explained and connected to leadership activities
- 3: the key concepts were accompanied by an activity in which the principal had the opportunity to practice the knowledge and/or skills of the EA.

The matrices of the analysis indicated almost all EAs were at least mentioned in the text, readings, or video clips comprising the AIM-SLP content. About one third of the EA subsections were addressed through an explicit, required activity. Such activities provided participants with opportunities to learn about the appropriate knowledge and skills through reflection or action (e.g., observing some of the AIM PD activities with participating teachers). For example, Activity C1 in Table 2 required participating principals to review the professional norms for examining teacher practice, view a video of classroom instruction, and use the project's Math Discourse Matrix (Sztajn et al., 2021) to analyze the nature of the discourse they observed. The activity asked them to consider what they noticed about the nature of student mathematics discourse, the teacher's instruction, and how the discourse shaped students' opportunities to learn the mathematics content.

EAs for NCSM Guiding Principle 1

The analysis showed AIM-SLP consistently provided strong opportunities for principals to learn and engage with EAs related to Guiding Principle 1. EA1.1, EA1.2, and EA1.3 received some of the highest and most consistent scores across multiple activities, particularly in AIM-SLP Module 3: A Vision of High-Quality Math Discourse for All Students. This finding indicates the program promotes a shared vision of HQMI and actively supports school leaders in developing the knowledge and practices necessary to foster this vision in their schools. Activities associated with these EAs frequently went beyond mentioning or explaining concepts by offering opportunities for leaders to practice and reflect, which was noted as a strength of the program.

EAs for NCSM Guiding Principle 2

EAs connected to Guiding Principle 2 showed varying levels of emphasis across the program. Although these components received lower scores in earlier modules, they demonstrated stronger alignment in Module 5: Preparing for Practice and Module 6: Supporting Teachers' Professional

Table 1*Structure of the NCSM Framework for Leadership in Mathematics Education*

Guiding principles	Big ideas	EA
1. Advocate and expect high-quality, equitable mathematics teaching and learning for every student	Curriculum leadership Instructional leadership	1.1: Ensure every teacher possesses a shared understanding and vision of HQMI and the actions required to meet the vision. 1.2: Ensure every teacher possesses the skills and knowledge necessary to design and implement meaningful learning experiences that lead to student understanding of mathematics. 1.3: Ensure all stakeholders have a clear understanding of HQMI and how to support it.
2. Design and implement structures that support high-quality mathematics teaching and learning by every teacher	Structures that support equity Structures supporting teachers' work with students Structures supporting teacher professional learning	2.1: Ensure mathematics learning for all students through organizational structures, time and resource allocation, and systemic supports that are aligned, intentional, and equitable. 2.2: Ensure systems of continual collaborative, job-embedded professional learning to build teacher and leaders capacity and increase efficacy. 2.3: Ensure sustainability through engaging all stakeholders in systemic, long-range strategic planning for all teaching and learning improvement initiatives.
3. Empower and nurture a culture of productive professionalism	Beliefs and mindsets Building relationships Culturally sustaining practices	3.1: Ensure assumptions, beliefs, expectations, and habits are examined to shape the school or department culture around teaching and learning of mathematics. 3.2: Ensure a culture of reflection, refinement, and action focused on continuous improvement in mathematical learning. 3.3: Ensure students, teachers, families, and community partnerships are built on meaningful engagement.
4. Monitor and act on evidence of student learning	Classroom level Teacher teams District level	4.1: Ensure the design and use of high-quality, aligned assessments and equitable assessment processes that guide meaningful reflection and action. 4.2: Ensure that the evidence of learning collected from every assessment is used to inform the design of curriculum, instruction, and the assessments themselves. 4.3: Ensure every student is provided access to grade-level content and intensification based on evidence of student learning.

Table 2*Snapshot of a Portion of the Analysis Spreadsheet*

EAs	Project AIM-SLP Module 3 components				
	Discourse	HQ		Analyze discourse	
	A1: Watch	B1: Read	B2: Reflect	C1: Examine	C2: Review
1.1: Ensure every teacher possesses a shared understanding and vision of HQMI and the actions required to meet the vision.	1	1	2	3	1
1.2: Ensure every teacher possesses the skills and knowledge necessary to design and implement meaningful learning experiences that lead to student understanding of mathematics.	1	1	2	3	1
1.3: Ensure all stakeholders have a clear understanding of HQMI and how to support it.	1	1	2	3	1

Note. From Module 3: A Vision of High-Quality Math Discourse for All Students.

Learning, where they consistently achieved ratings of 2–3 across multiple activities. This pattern suggests an intentional program design that first establishes instructional leadership foundations before transitioning to the organizational structures needed to sustain improvement. The higher ratings in Modules 5 and 6 indicate that AIM-SLP addresses these critical systemic elements, particularly as participants move toward practical implementation in their school contexts.

EAs for NCSM Guiding Principles 3 and 4

EAs associated with Guiding Principles 3 and 4, including cultivating beliefs, culturally sustaining practices (i.e., EA3.1 and EA3.2) and leveraging assessment data to monitor students' learning (i.e., EA4.1 and EA4.2), were less frequently and deeply addressed across program activities. Although some elements related to reflection (e.g., EA3.1 through ways for collaborating with mathematics coaches and teachers to build reflective culture) and culture building (e.g., EA3.3 through emphasizing importance of communicating with caregivers and sharing caregiver newsletter resources) were present in the activities, there were limited opportunities for principals to actively practice strategies and use the tools in these areas. One particular set of EAs was somewhat problematic to judge in terms of alignment with the AIM-SLP, namely the following EAs under GP4: Monitor and act on evidence of student learning:

- EA4.1: Ensure the design and use of high-quality, aligned assessments and equitable assessment processes that guide meaningful reflection and action.
- EA4.2: Ensure the evidence of learning collected from every assessment is used to inform the design of curriculum, instruction, and the assessments themselves.

The text in the NCSM leadership documents focuses primarily on formal and summative assessments rather than the very formative assessments emphasized in Project AIM for initiating and supporting classroom discourse in K–2 mathematics lessons. Specifically, formative assessment is a core element of Project AIM because it informs the teachers' moment-by-moment instructional decisions about classroom discourse. Accordingly, the Project AIM evaluator concluded, the lack of evidence for EA4.1 and EA4.2 should not be construed as a critique or weakness of the AIM-SLP. Rather, it is appropriate that the formative assessment in the AIM-SLP addresses providing every student with access and practices aligned to in-the-moment instructional decision making and proximal steps of classroom instruction. In fact, the third EA under GP4, EA4.3: Ensure every student is provided access to grade-level content and intensification based on evidence of student learning, was mirrored throughout the AIM-SLP activities and was therefore judged to align well with the instructional elements provided to participating principals.

Study 3: Principals' Perspectives

Study 3 aimed to answer the third research question: What are the perspectives of participating principals regarding the contributions of the AIM-SLP to their support for high-quality mathematics discourse? Through the analysis

of principals' perspectives, we explored how AIM-SLP experiences contribute to their practices to lead, support, and sustain high-quality mathematics discourse in their school settings.

Two principals, Ayanna and Sarah, engaged in the AIM-SLP beginning in Summer 2023 and continuing throughout the 2023–2024 school year. Ayanna and Sarah worked for the same school district in the southeastern region of the United States. The district was in a small city setting with 11 elementary schools and served nearly 12,000 students, of which 54% were from racially/ethnically underrepresented populations and 48% were economically disadvantaged. Ayanna had been the principal at the same school for 7 years. Ayanna's school was a PK–5 public elementary school serving nearly 350 students, of which 44% were from racially/ethnically underrepresented populations, and 31% were economically disadvantaged. Sarah had been her school's principal for 9 years. It was also a PK–5 public elementary school serving a slightly larger population of nearly 400 students, of which 54% were from racially/ethnically underrepresented populations and 48% were economically disadvantaged. Both Ayanna and Sarah had been assistant principals of their school prior to becoming the head principals.

To capture principals' perspectives and experiences participating in AIM-SLP, we collected Ayanna's and Sarah's written responses to key SLP activities documented in the learning management system and interviewed each principal three times spanning their participation (initial, follow-up, and final). The process was as follows:

- The initial interview focused on principals' early impressions, key takeaways from the first half of the program, and feedback on specific activities and tools in Modules 1–5. This interview lasted 30–55 minutes.
- The follow-up interview, conducted at the end of the 1st semester of the 2023–2024 academic year, explored principals' learning about leadership practices from Modules 6–8. It elicited their successes and challenges in applying ideas and practices from AIM-SLP modules in their schools and lasted 30–45 minutes.
- The final interview, conducted at the end of the 2nd semester of the 2023–2024 academic year, provided principals with an opportunity to reflect on the AIM-SLP's impact on their leadership practices and knowledge. It lasted 20–45 minutes.

We employed a thematic analysis approach drawing from multiple data sources (i.e., transcripts of interviews with the principals and their responses to key activities in AIM-SLP). We used a combined technique of inductive and deductive thematic analysis in a collaborative coding process (Fereday & Muir-Cochrane, 2006; Richards & Hemphill, 2018). The deductive data analysis was guided by selected practices from the unified model of effective leadership practices (Hitt & Tucker, 2016) to explore how AIM-SLP supported these practices. Meanwhile, two researchers conducted an inductive analysis in areas where the framework did not fully capture principals' perspectives. Any discrepancies in coding were resolved through discussion until consensus

was reached. Table 3 presents the final coding scheme for the study.

Table 3

Coding scheme: Effective Leadership Practices

Practice domain	Practices
Establishing and conveying the vision*	Communicating broadly the state of the vision Setting expectations and goals*
Building professional capacity	Providing opportunities to learn Engendering responsibility for learning
Creating a supportive organization for learning	Strengthening and optimizing school culture Sharing and distributing leadership
Teaching and learning environment	Providing instructional support: • Having targeted and reflective conversation** • Intentional observations and walkthroughs** Developing a shared language***

Note. * Slightly revised language from original practice in the framework. ** Subpractice that is coded as a result of inductive analysis. *** Practice that is coded as a result of inductive analysis.

Our analysis showed AIM-SLP supported participating principals in four main leadership practice domains: (a) establishing and conveying the vision of HQMI, (b) building professional capacity, (c) creating a supportive organization for learning, and (d) enhancing the teaching and learning environment.

Establishing and Conveying the Vision of HQMI

Principals described how their participation in the AIM-SLP influenced their understanding about establishing and conveying the vision of high-quality mathematics discourse across grade levels. They shared their vision of sustaining AIM ideas (e.g., mathematics talk techniques), particularly as students transitioned from K–2 to Grades 3–5. The principals expressed the need for deliberate planning to ensure discourse practices developed in early grades would not be lost in subsequent years. Both principals highlighted the value of Project AIM's *Activating Math Talk* book (Sztajn et al., 2021) as a tangible resource for supporting consistent practices across grade levels.

Principals' takeaways arising from their work in the AIM-SLP also emphasized the importance of setting clear expectations and goals for high-quality mathematics discourse. They highlighted fostering meaningful student engagement through student-led discussions, selecting discourse promoting tasks, and supporting responsive discourse (Sztajn et al., 2021) in mathematics classrooms as critical expectations. Principals recognized the need

to ensure such practices become embedded daily in mathematics teaching. For instance, Ayanna noted:

I always say to my staff, whoever's doing the talking is doing the learning. If you walk into my math classes, you'll see kids talking. . . . How do you have kids adding on and really leading the discussion? . . . These strategies [referring to AIM discourse techniques] are part of the expectation. What we've learned in AIM should show up in lesson plans.

Building Professional Capacity

Analysis of principals' interview and program reflection data revealed two types of practices for providing opportunities to learn. One practice involved principals focusing on their own learning as instructional leaders. They shared AIM-SLP's expectation that principals participate in parts of the AIM teacher PD sessions alongside their teachers to help them learn ideas (e.g., AIM's mathematics discourse techniques and discourse types) that support instructional leadership. For instance, Sarah articulated, "Whenever we're learning something new, that 1st year, I go through the learning process with my staff. I do it all with them. The 2nd year, I start giving feedback and implementing because I feel more equipped."

The principals also explained their plans for expanding and sustaining opportunities for professional learning for their teachers related to AIM ideas in their schools. For instance, Sarah shared:

The biggest takeaway has been having time and space for people to reflect on their practice, not just by yourself, but as a group, as a team. To do a thing well—to get information about it, reflect on it, and then engage in dialogue—helps us move forward.

Principals also noted that AIM-SLP supported them in cultivating a growth mindset among teachers while engaging with AIM-related learning opportunities. They especially acknowledged their role in engendering responsibility for learning by encouraging and creating spaces for teachers to implement new mathematics talk techniques in their classrooms. From her experience, Ayanna recounted:

Anytime there's something new, I think there's this uneasiness like, "Am I doing it right?" And I always tell them, "You're not gonna harm the kids trying. Harming kids is not trying something or not teaching them." When they first do a new technique, they feel uneasy. . . . We talk about being uncomfortable . . . and it's okay. 'Cause that's when the learning takes place.

Creating a Supportive Organization for Learning

Both principals, Ayanna and Sarah, said after participating in the AIM-SLP, they focused on strengthening and optimizing school culture to create a coherent approach to high-quality mathematics discourse across grade levels. They indicated their plans for several professional learning opportunities to ensure AIM-related learning reached teachers across K–5 grade levels (e.g., coaching cycles, book clubs focusing

on *Activating Math Talk*, peer observations). They aspired to reinforce and integrate AIM concepts schoolwide, ultimately establishing a consistent and effective approach to mathematics discourse in their schools' classroom culture.

Sarah appreciated how AIM-SLP influenced her leadership approach, shifting from a general focus on school culture to a more intentional emphasis on continuous improvement and targeted growth areas. She elaborated:

What are we going to do as a school? Through coaching cycles, how do we apply this? What does it mean for third, fourth, and fifth grades as we onboard them with the information? . . . It's about narrowing focus on a culture of improvement rather than just overall school culture.

Sarah also identified strategies for strengthening school culture by fostering engagement with parents and caregivers (e.g., incorporating AIM-SLP resources for family mathematics night activities, using caregiver newsletters to promote ongoing communication and productive mathematics talk at home).

The principals also acknowledged that creating school culture requires a collaborative effort. They found the AIM-SLP supported them in using strategies for distributing leadership responsibilities among staff (e.g., coaches, teachers). Ayanna described her plans to use specific tools and resources from the AIM-SLP to share leadership responsibilities with her school's instructional coach. Sarah specified her plans, saying:

It's myself, coaches, and some key person in the grade levels. One of the other things that's going to be quite instrumental is having a teacher on each of the grade levels who has experienced the AIM work and is excited to share the work. While the coach is going to be coaching the grade level, having the teacher also take leadership roles.

Enhancing the Teaching and Learning Environment

The principals shared the AIM-SLP supported them in three specific ways to provide instructional support for their teachers to enhance the teaching and learning environment. First, they started to engage in more targeted conversations with teachers during lesson planning, lesson observation reflections, and post-PD reflections. They used AIM's conversation guides, the Questions to Ask Yourself resource, and the Math Discourse Matrix (Sztajn et al., 2021) to ask specific questions and identify key ideas to push teachers' thinking and improve instruction. For instance, Sarah noted, "[The AIM-SLP] helps me to push thinking and to be able to ask a more specific question. So, being able to have very targeted questions or targeted look fors."

The principals also shared their intention to use AIM-SLP's tools and resources to engage in more targeted conversations—not only with teachers, but also with instructional coaches. For example, Sarah began incorporating conversation guides into her meetings with coaches and said, "Now, I feel more targeted in my conversations because, earlier, some of my conversations have been around what's been happening, but this just gave me more knowledge."

Second, Sarah and Ayanna emphasized that AIM-SLP supports them in conducting intentional classroom observations aimed at identifying specific areas for teacher growth. They recognized AIM-SLP resources (e.g., conversation guides, Questions to Ask Yourself), along with the discourse techniques, were helpful for the observations. Both principals also highlighted that AIM-SLP supports them in conducting intentional walkthroughs that move beyond broad reflections like, "What went well." Sarah and Ayanna planned to use the AIM-SLP's walkthrough checklist tool to gather key information about instructional practices and student discourse. They intended to use this information to provide specific instructional support.

Both principals acknowledged that developing a shared language is essential for providing intentional instructional support for teachers and fostering a collaborative school culture. They emphasized AIM-SLP's encouragement for principals to actively participate in AIM-PD with teachers helped them establish this shared language. Ayanna reflected:

So, I do think having some of that common language, so, when we're in a meeting in PLCs [professional learning communities], if they're not talking about it, I can remind them. When I'm doing observations, I can just say, "Hey, this could have been a time where you used this strategy."

Additionally, Sarah noted the AIM-SLP gave her the knowledge and confidence to engage in productive instructional conversations with teachers. She was better able to connect with teachers' experiences by developing shared language, which strengthened her instructional leadership, built trust, and increased teacher buy-in.

DISCUSSION AND IMPLICATIONS

This study addressed the need for mathematics-focused principal PD by designing the asynchronous, job-embedded AIM-SLP aligned to teacher learning (Kazemi & Resnick, 2020) and aimed at supporting school leaders in fostering high-quality mathematics discourse as a key component of HQMI (Munter, 2014). The AIM-SLP offers an intentional model for school leaders' professional learning grounded in four design principles aligned with the well-established framework for leadership in mathematics education (NCSM, 2020) and supported by early evidence of relevance and usability among school principals.

The findings from three focused studies suggest: (a) strong alignment between principals' expressed needs and the AIM-SLP design principles; (b) consistent alignment of AIM-SLP activities with NCSM EAs for GP1 and GP2, with lighter emphasis on EAs for GP3 and GP4; and (c) support throughout AIM-SLP for leadership practices in four domains. Collectively these results suggest AIM-SLP is a promising approach for supporting school leaders to cultivate high-quality mathematics discourse as a key component of HQMI.

The results from these studies also carry important implications for those designing PD for school leaders. One implication is the importance of creating intentional

opportunities for leaders and teachers to learn together. Alignment between AIM-PD and AIM-SLP creates a powerful, shared learning model. When principals engage with the same tools and language as their teachers, their instructional support becomes more relevant and connected to classroom practices. This shared learning not only strengthens the coherence of PD across teachers and school leaders, but also allows leaders and teachers to work toward a shared vision and goals of instructional improvement in mathematics. In addition, the development of shared language extends beyond strengthening instructional feedback or recognizing high-quality mathematics discourse in K–2 classrooms for school leaders. Shared language fosters a culture where teachers, coaches, and school leaders can all contribute to schoolwide improvement and encourages broader engagement in promoting discourse-rich mathematics instruction. It also supports a clear instructional vision with high expectations that everyone can understand and uphold.

The second implication is the value of the tools that are job-embedded and easy to enact. School leaders need resources that help them notice, reflect, and respond to HQMI in real time. Practical tools (e.g., conversation guides, walkthrough checklists) shared in the AIM-SLP served as boundary objects that enabled principals to ground their conversations, observations, and feedback in a common framework. These tools became catalysts for principals moving from knowing to doing.

A third implication is that school leaders' appreciation of high-quality mathematics discourse as a key aspect of HQMI, developed through participation in AIM-SLP, prompted

them to plan for intentional and coherent scale-up from K–2 classrooms to K–5. A recommendation for future design of school leader PD programs, including AIM-SLP, is to include extensions that support leaders in building a coherent instructional vision and discourse practices spanning all elementary grades in the school setting. This could be an important first step toward creating coherence from the classroom to the district office for instructional improvement (Cobb et al., 2020).

One final implication relates to the partial alignment between AIM-SLP and EAs in NCSM's (2020) framework for leadership in mathematics education, particularly around formal assessment. This finding highlights the broader point that not every PD program should be expected to address all aspects of a comprehensive leadership framework. Rather than treating this as a limitation, our findings suggest a more nuanced view: frameworks are guides, not checklists. Embracing this perspective invites future researchers and PD designers to consider more adaptive approaches to both program development and evaluation, where alignment is examined thoughtfully in ways that capture the unique goals and contexts of each PD model.

The three studies we conducted are exploratory and have limitations. In particular, only two principals engaged in the AIM-SLP, and the evidence from the third study relied heavily on self-report of the principals' intended, rather than enacted, practice. In future research, we will examine how leaders' participation in AIM-SLP translates into realized changes in their leadership practice and, ultimately, into classroom instruction and student learning.

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A CASE OF HOW PROFESSIONAL DEVELOPMENT CAN HELP COACHES SUPPORT TEACHERS IN PROMOTING CLASSROOM MATHEMATICAL ARGUMENTATION FOR ALL STUDENTS

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ABSTRACT

This article overviews how professional development (PD) can help coaches support teachers in fostering classroom mathematical argumentation (CMA) for all students. After summarizing the conceptual underpinnings of a PD program on coaching, the article describes the program, presenting a case study of how structures and tools designed by the PD for lesson planning and reflection informed a coach's discourse with a teacher. Analyses of transcripts showed the extent to which a coach and a teacher discussed various topics related to mathematical concepts and practices and students' participation during early and late coaching cycles, informed by the purposes and protocols designed to visualize, plan, and debrief lesson enactment. The article highlights recommendations for PD design and research to support mathematics coaches.

Keywords: professional development, coaching, mathematical argumentation, qualitative research, case study

Since the late 1980s, prominent professional organizations (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSSO], 2010) have proposed ambitious goals for all students' learning of mathematics. These goals prioritize conceptual understanding and procedural proficiency in mathematics, constructing mathematical arguments in learning mathematics, communicating mathematical ideas to others, and using a variety of representations (NCTM, 2000; NGA Center & CCSSO, 2010). These organizations also emphasize fostering a positive attitude and confidence in mathematics among students.

Furthermore, every student should have "access to rigorous mathematics that develops their understanding, skills and

knowledge, along with the confidence to leverage their learning" (National Council of Supervisors of Mathematics, 2024, para. 5) and improve their world. NCTM (2014) further emphasized teaching practices that facilitate classroom discourse and support students in analyzing and comparing mathematical arguments. Aligned with these student learning goals are various models of teachers' professional development (PD), including coaching, which research has shown can have positive effects on teaching practices and students' assessment scores (further discussed later in this paper), particularly with highly trained coaches (Campbell & Malkus, 2011). In fact, significant policy-related publications authored by NCTM (2020) have advocated for coaching as a means for improving student learning.

Although a coach's assigned duties may include managing assessments and other administrative tasks (Campbell & Griffin, 2017), key coaching activities take place in the teacher-coach dyad (i.e., two people who work together with a common purpose). This article presents a case of how a coach drew on innovative structures and tools designed by the Visualize Teaching Professional Development (VTPD) program to support a teacher in promoting classroom mathematical argumentation (CMA) for all students. The case study addressed the following research question: How do PD structures and tools designed for lesson planning and reflection inform the topics of a coach's discourse with a teacher in promoting classroom mathematical argumentation for all students?

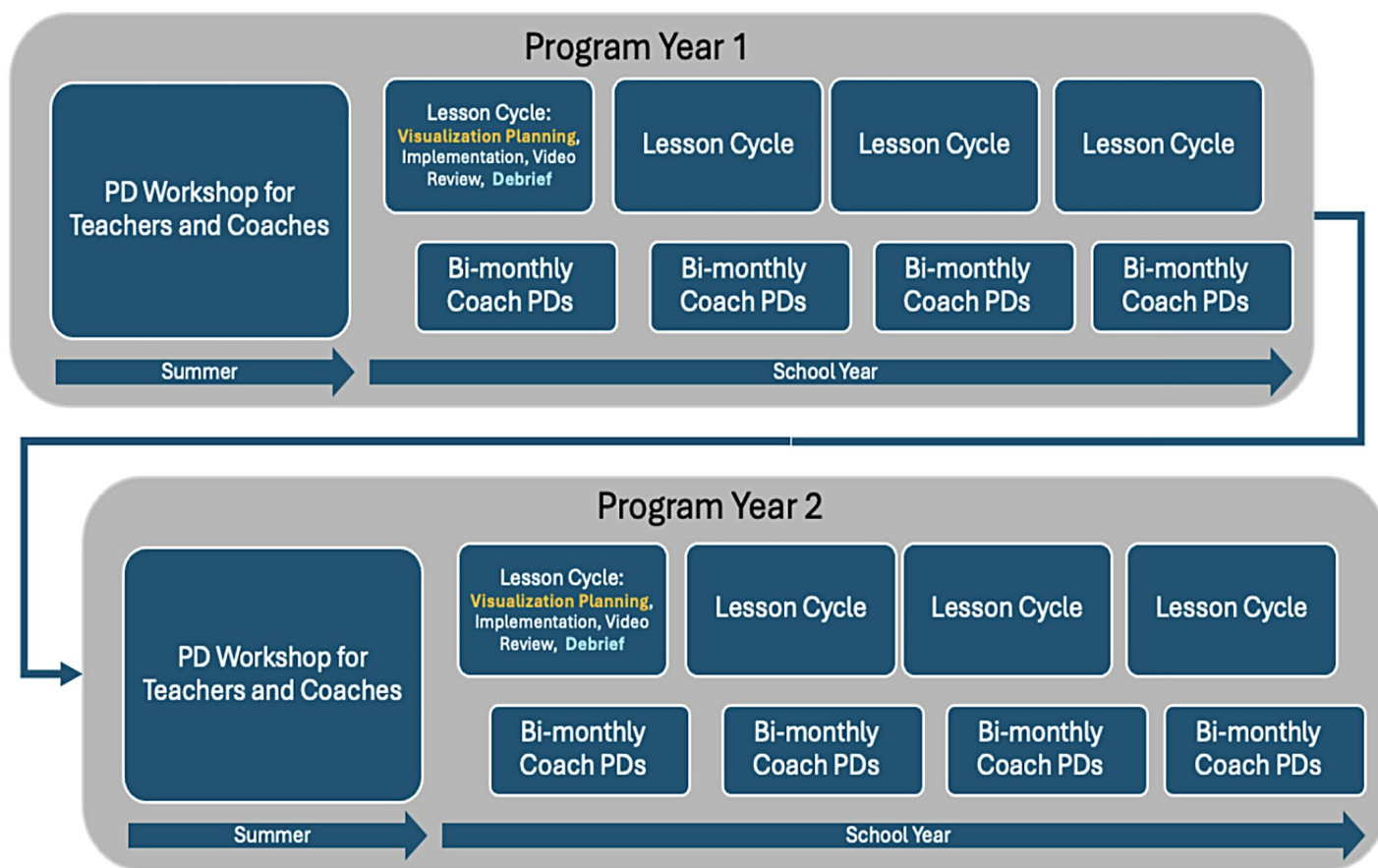
We describe the VTPD program in the next section, highlighting its conceptual underpinnings and associated structures and tools for coaches and teachers. This description is followed by a review of relevant literature on coaching. Next, the research methods are explained. The results show portions of the VTPD program in use in a coach's one-on-one practice with a teacher, with evidence highlighting how the coach took up structures and tools in a meaningful way in her interactions with the teacher. Finally, the article discusses limitations of the study and offers recommendations for PD design and research to support mathematics coaches.

Overview of the VTPD Program

The VTPD program focuses on teaching and learning mathematics argumentation, a high-level disciplinary practice that is important for all students to learn (Knudsen et al., 2018). Its goals are that teachers acquire (a) the ability

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Figure 1
Content Foundation: CMA



to make mathematical arguments for themselves and (b) a basic set of teaching moves to support every student in making their own arguments. The 2-year program includes (a) two 4-day summer workshops; (b) coach–teacher dyads engaging in the recommended number of four to six coaching cycles, each incorporating specially designed lessons, throughout 2 school years; and (c) monthly meetings with project staff to support coaches (see Figure 1). This study is situated in the coaching cycle.

Any math-focused coaching is influenced by consideration of what mathematics is important to learn (Martin & Harel, 1989); thus, the mathematics VTPD addresses is critical to the nature of what coaches and teachers do together. The major content focus of VTPD is mathematical argumentation in the classroom, which is emphasized in many current national and state standards (e.g., National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). VTPD is designed to support CMA, which is defined as a social process in which students and their teacher discover the mathematical truth together (Knudsen et al., 2018).

Over the first years of the VTPD team’s work with teachers, which preceded the current study, the team designed a four-part model of argumentation collaboratively with teachers new to this mathematical practice to support their own learning and use in their classrooms. The model includes stages of conjecturing (i.e., making bold claims based on evidence), justifying (i.e., establishing the truth or falsity of

conjectures), concluding (i.e., coming to common agreement on the truth status of the conjecture), and generating cases (i.e., finding examples to suggest patterns that can yield conjectures; see Figure 2). VTPD is grounded in prior research on the structure of mathematical argumentation (Harel & Sowder, 2007; Krummheuer, 1995). CMA and the four-part model are important to understanding the coach–teacher discourse examined in this study.

Figure 2
Four-Part Model of CMA

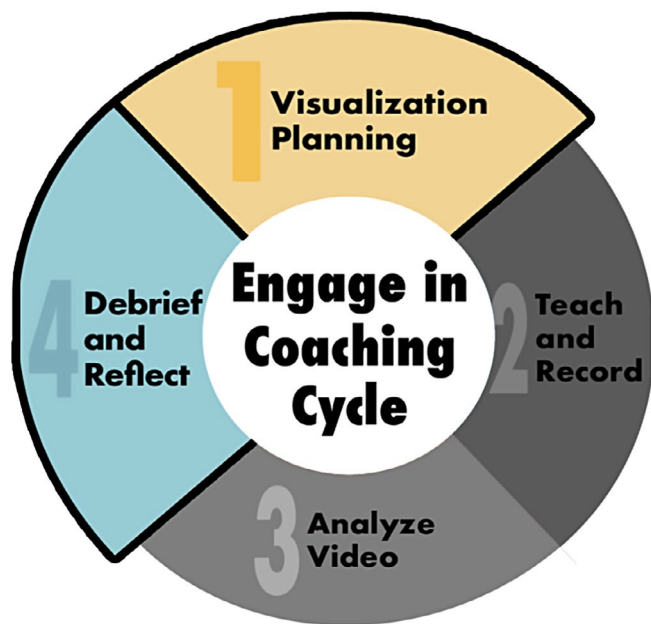


The Coaching Cycle

VTPD's coaching cycle consists of plan, teach, analyze, and debrief (or reflection) phases (see Figure 3 for overview of coaching cycle; Phases 1 and 4, as highlighted in this figure, were the foci of the study). Protocols, available for each phase of the four-part coaching cycle, guide coach–teacher dyads in structuring their time together.

Figure 3

Four-Phase Coaching Cycle



To prepare for visualization planning (VP), in Phase 1 of the cycle, the teacher selects a lesson designed to support CMA. Not every math lesson, even those that address conceptual understanding, affords students opportunities to make conjectures and justify them. VTPD offers focal lessons, some of which are based on lessons from Illustrative Mathematics (IM, 2021), an open-access curriculum available online to schools and teachers worldwide, and all of which address grade-level standards. The lessons are modified to create those affordances.

VP is designed to direct coaches' and teachers' attention to students' arguments and teaching moves that support them. Figures 4 and 5¹ show a protocol designed to structure coaching routines for VP. The teacher uses a combination of imagination and experience to envision how this lesson might go with her students as she talks through the lesson. First, the teacher and coach complete the mathematics of the chosen lesson together, making mathematical arguments for themselves, an important precursor to what comes next. Then, they map out several different routes students may take to make arguments. The teacher also plans to address the needs of target students who are chosen because they rarely participate in whole-class discussions. In the dyad, the coach is the active listener, probing the teacher with questions, offering insights, and writing a lesson plan as the visualization proceeds.

Figure 4

VP Protocol: Teachers

Visualization Planning: Teacher's Role

01

Prepare the focal lesson

Select one of the lessons developed by the Visualization Teaching team. Review the lesson and begin considering how you will emphasize math argumentation for student learning.

02

Meet with your coach

Schedule at least 30 minutes to 1 hour with your coach to review your prepared focal lesson, allowing time to "do the math together" before discussing how you will teach the lesson.

03

Do the math together

Before discussing your focal lesson, work through the math problem together. Take this time to consider the different ways your students may answer the math problem.

04

Engage in Visualization Planning

Imagine what teacher moves you will make and how students respond. Go through as many "what if" scenarios as possible. For example, if you ask one student to explain their idea, but the other students don't understand it. What will you do next?

05

Schedule your debrief

After your visualization planning, ensure you schedule your debrief within 1 week of teaching your lesson. Set aside another 30 minutes to 1 hour to discuss a specific segment of your recorded lesson, utilizing the Discussion Guide to structure your debrief.

Figure 5

VP Protocol: Coaches

Visualization Planning: Coach's Role

01

Review the lesson

Before meeting with your teacher, review the selected focal lesson to adequately prepare yourself for discussing teacher and student moves to support your teacher through the visualization planning.

02

Do the math together

Before discussing the focal lesson, work through the math problem together. Take this time to consider the different ways your teacher's students may answer the math problem.

03

Engage in Visualization Planning

Support your teacher in imagining their teacher moves and how they will support student learning. Ask clarifying or probing questions, ensuring your teacher considers several possible scenarios for student learning.

04

Take Notes

As you go through the Visualization Planning session, ensure you are taking notes and filling out the template to maintain a record of the teacher's moves and possible student responses.

05

Schedule your debrief

After visualization planning, ensure you and your teacher have scheduled the debrief within 1 week of teaching the lesson. Set aside another 30 minutes to 1 hour to discuss a specific segment of the recorded lesson, utilizing the Discussion Guide to structure your debrief.

¹Figures 4–6 and the protocols depicted in them were developed by the VTPD team.

In Phases 2 and 3, the teacher uses the lesson plan, devised during the VP session, to teach and record the lesson. This video recording of the lesson is then uploaded to a cloud-based system, offering teachers, coaches, and researchers a simpler way to view the recordings, to which the teacher and coach can add commentary directly. This recording is the basis for Phase 3. The teacher chooses a segment of the video for analysis, and then the teacher and the coach view and annotate video time points independently. These annotations become topics of coach–teacher dialogue in Phase 4.

In Phase 4 (i.e., debrief), the coach and teacher engage in evidence-based discourse facilitated by the discussion guide (see Figure 6). This guide is a rubric of characteristics of good teaching practice for mathematical argumentation, derived from prior literature. Three characteristics are (a) maintaining cognitive demand of questions, (b) orchestrating classroom discourse, and (c) ensuring access to mathematical concepts and practices for every student (e.g., Kuchemann & Hoyles, 2009; Otten et al., 2015; Selling, 2016). Teachers, with coach support, rate their teaching for the target lesson in each of these categories. The guide also provides characteristics of productive student discourse for teachers to rate for the same lesson (see Figure 6 for details regarding instructions on rating teaching and student moves) and setting goals for the next lesson within the coaching cycle. Certain levels may not be clearly distinguishable. Teachers may decide to use only one or two of the levels for a particular category.

Additional Learning Activities for Coaches

In addition to participating in the 4-day summer workshops with their teachers, coaches have other learning opportunities with VTPD (e.g., 2 coach-specific workshop days bracketing the 4-day workshops with teachers). These 2 days address areas of interest to coaches (e.g., characteristics of adult learners, how the VTPD coaching model fits with their current coaching practice). During the 4-day CMA workshops with their teachers, coaches fortify existing relationships and are given opportunities to work in teacher–coach dyads to focus more deeply on their learning and see each other as colearners of argumentation. During the academic year, coaches work with their teachers through multiple coaching cycles. Coaches, on their own, also participate in bimonthly 1.5-hour virtual professional learning community meetings with program staff and other VTPD coaches from around the United States. VTPD coach meetings generally consist of a short social check-in, a mathematical argumentation activity, and discussions of coaches' VP or debrief sessions with teachers to spark further dialogue.

Supporting Each and Every Student

The goal of reaching each and every student is core to contemporary teacher PD (Nasir & Cobb, 2002; NCTM, 2000, 2014). VTPD is designed to foster all students' participation in CMA through teachers' use of moves in the three categories from the discussion guide (i.e., cognitive demand, orchestration, access) along with insights they gain from the summer workshop. As students become more comfortable making "bold conjectures" (Knudsen et al., 2018) and putting together justifications that are convincing to themselves and

others, they can become more active participants in class and more engaged in learning.

Teachers and coaches have ongoing opportunities to learn to use mathematics argumentation to promote classroom discourse. As a basis for CMA, teachers can choose from VTPD argumentation-focused lessons aligned with the standards teachers must address and typical curricular sequences to ensure CMA is not an add-on. During coaching cycles, the protocols and structures described previously continue to emphasize full student participation in CMA. In the debrief phase, the discussion guide provides support to reflect on the success of those strategies through analyses of both teacher and student moves.

LITERATURE REVIEW

Coaching Defined

VTPD coaches may be called disciplinary (i.e., content) or instructional coaches. A content-specific approach to coaching emphasizes the "students' learning of a particular subject, and teachers' plans, strategies, and methods are discussed in that context" (Mudzimiri et al., 2014, p. 4). In this regard, the student's mathematical learning is the primary focus of the coaching session (Mudzimiri et al., 2014). In contrast, the instructional coach focuses on a "broad range of instructional issues, which might include classroom management, content enhancement, specific teaching practices, formative assessment" (Knight, 2009, p. 31).

For this article, we considered VTPD coaches as engaged in a third type of coaching that includes elements of the two coaching types named previously—math instructional coaching. We adopted the definition by Kane and Saclarides (2022): "A mathematical instructional coach is an educator who has deep mathematical pedagogical knowledge and who works closely with classroom teachers to improve student learning in mathematics" (p. 8). In the rest of this article, we refer to VTPD coaches as "math coaches" or simply "coaches."

What Coaches Need to Know

Researchers have identified broadly what mathematics coaches need to know. Common categories include (a) mathematics knowledge for teaching, (b) leadership skills, (c) interpersonal skills, (d) knowledge of when to use specific coaching activities, and (e) knowledge of the practices they guide teachers from and toward (Coburn & Russell, 2008; Gibbons & Cobb, 2016; Kane & Saclarides, 2022). However, a focus on coaches' knowledge of how to promote full student participation has rarely been addressed in research on coaching (Marshall & Buenrostro, 2021), an issue central to VTPD coaching tools and protocols. Nonetheless, there are resources available for practitioners (e.g., Bennett, 2014; Civil et al., 2019; Nasir & Cobb, 2002; Secada, 1994; van Es et al., 2022). A promising resource is Aguilar's (2020) book, which offers practical advice for coaches with full student participation as the centerpiece. The project team provided this book, and distillations of literature in its tools and resources, to VTPD coaches.

Figure 6
Debrief Meeting Discussion Guide

Student Moves

Math Practice 3 - Construct arguments and critique the reasoning of others

Conjecturing

Level 1	Level 3
Students make specific conjectures.	Students make generalized conjectures.
When comparing car speeds with lines of different slopes, students state, "The blue car goes faster."	Students say: "the steeper the line, the faster a car travels."
Students give the answer to a linear equation with one variable.	Students conjecture about the method for solving <i>all</i> linear equations with one variable.

Justifying

Level 1	Level 2	Level 3
In trying to justify, students simply repeat the conjecture, cite authorities such as the teacher or textbook, or rely on visual information.	Students use cases (empirical). Students use a finite number of cases to support a specific or generalized conjecture.	Students use generic cases or general properties to support a generalized conjecture.
To justify that the sum of any two even numbers is even, students may say that last year's teacher told them that.	Students conjecture that the sum of any two even numbers is even, Then try $2+4$, and $4+8$, and stop there because "they work."	Students show with counters that pairs of even numbers have no leftovers. They say, "If you combine two even numbers there won't be any leftovers, so the sum is even."

Math Practice 5 - Using appropriate tools strategically

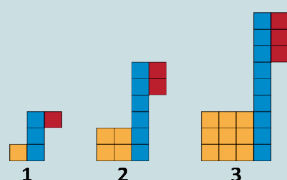
Level 1	Level 3
Students use tool procedurally under direct instruction by the teacher.	Students use a tool independent of direct instruction to express or explore an idea, generate cases, form conjecture, or make a justification
Students learn how to plot points on a coordinate graph following the teacher's instructions.	Students build a table to organize data or plot points in graphs to see patterns. Students use a shared diagram displayed by the teacher to communicate or clarify their conjecture/justification.

Math Practice 6 - Attend to precision

Level 1	Level 2	Level 3
Students or teacher ask for clarity or greater precision, AND	Students or teacher asks for clarity or greater precision, AND	Students or teacher asks for clarity or greater precision, AND
No attempt is made to further precision.	Students attempt to further precision but do not actually do so.	Students actually do further precision.
The teacher tells students their work is incorrect but offers no help or guidance, and the student does not try to correct it.	A student asks for clarification of a conjecture and a student re-words it in more precise mathematical language, but other students do not seem to understand the conjecture any better.	Students disagree with specific statements or imprecise calculations, and they offer clarification or correct calculations. Students refine a conjecture or explanation when someone says they don't understand it or don't think it addresses all cases and this helps the argument continue.

Math Practice 7 & 8 - Reason structurally

Level 1	Level 2	Level 3
Students focus on specific cases (of calculation, of solving an equation, of geometric shapes), without referring to patterns or general properties.	Students notice some patterns but without talking about the structure of those patterns (in calculations, expressions, and geometric shapes.)	Students focus on the relations between parts and the whole (i.e., structure) in number, expressions, or geometric shapes, or refer to general properties when conjecturing or justifying.



In finding a general pattern for any number n , students draw a shape for $n = 5$, and find the number blocks by counting individual blocks with no apparent systemic method.

Students say the pattern number is the same as the red blocks, but do not go further to see the effect on the overall growing pattern.

Students describe that blue blocks increase by 3 as the step number increases by 1 and expresses it generally, by saying “ $3n$ for any n .”

Students decompose a shape into rectangles and triangles and use their properties to find the area of the whole shape.

Teaching Moves

Maintenance of Cognitive Demand

If the teacher introduces a high-cognitive demand task, was the demand maintained?

Level 1	Level 2	Level 3
The teacher mostly asks questions so that students are mostly providing facts or procedures. Student turns are very short and directly respond to the teacher.	The teacher starts with a high-demand task, and starts out maintaining that demand, perhaps with a “why” question, but follow up questions mostly elicit facts or procedures only.	The teacher mostly asks questions that elicit student reasoning or engagement in the math practices and maintains this to the end of the task.
Asking “if we add these two numbers, what do we get?”	Asking, “Why does this algorithm work?” Then stating, “First, let’s make a rectangle. How should we divide it up?”	Asking students “why” questions that enable them to make their own justifications in support of a conjecture.

Orchestration

Does the teacher provide an opportunity for students to respond to each other’s ideas and interpret their peers’ ideas?

Level 1	Level 2	Level 3
The teacher rarely invites students to talk to each other. Discourse is mostly teacher-to-a-single-student.	The teacher encourages or invites students to respond to each other’s ideas, but often steps in to provide interpretation or additional questions between student turns. <i>Vertical discourse (t-s1-t-s2-t-s1)</i>	The teacher often encourages or invites students to respond to each other’s ideas, directly to each other.
<i>Vertical discourse (t-s-t)</i> Asking “OK, Darrel, how do you know your conjecture is true?” and sticking with Darrel.	“Thank you for your contribution. What do others think?”	<i>Horizontal discourse (t-s1-s2-s3-t)</i> Getting students to respond to each other with questions, answers, and offers.

Access

To what extent does the teacher support access to the task for each student, with particular attention to those who are typically left out?

Level 1	Level 2	Level 3
The teacher makes a broad invitation to all students to participate, but there is no follow up to engage students who are not part of classroom discourse.	The teacher makes an invitation to broaden participation in classroom discourse, but it is generic.	The teacher’s invitations to broaden participation are tailored to include those who have not yet been part of classroom discourse.
Asking “Who would like to make a conjecture?” and sticking with the one or two students who respond.	Asking “Does anyone else want to conjecture?”	Asking a student who is less confident in argumentation, “What do you think is false?” Then, “What do you think might be true?”

Progress Tracker

This is how you're doing on Teaching Moves and Student Moves

Cycle	Teaching Moves			Student Moves				
				MP3		MP5	MP6	MP7&8
	Cognitive Demand	Orchestration	Access	Conjecturing	Justifying	Strategic Tool Use	Attend to Precision	Reason Structurally
1								
2								
3								
4								
5								
6								

Reflections

Reflections on patterns of participation:

The Coaching Cycle

Many coaching programs are structured around a coaching cycle with at least three elements: planning, teaching, and reflecting on a lesson (Carey et al., 2011). Each element can be thought of as containing a set of routines. Gibbons and Okun (2023) asserted coaching routines can “manage the complexities of ambitious teaching collectively and raise questions or issues for further reflection” (p. 22). For example, Amador et al.’s (2023) study of coaching routines based on annotations of classroom videos found coaches’ practices shifted from summarizing aspects of the lesson to taking a more interpretative stance. Such research provided context for this study of the VTPD program, with an eye toward shifts in coaching practice to better support teachers.

Effectiveness of Coaching

A meta-analysis of 60 studies of coaching found significant improvement in a variety of teaching practices and measures of student achievement, including state assessments, National Assessment of Educational Progress scores, and researcher-made assessments (Harbour et al., 2018; Hill & Ball, 2004; Obara & Sloan, 2009). However, most of these studies were on literacy coaching, with only a few on mathematics coaching (Kraft et al., 2018). Two recent studies of large-scale mathematics coaching programs found significant positive effects on teachers’ classroom practices that promote students’ conceptual understanding and problem-solving skills (Kraft & Hill, 2020; Stein et al., 2022). These studies revealed models of mathematics coaching that are closely related to VTPD coaching. The studies, investigated, respectively, the planning and reflecting stages of a coaching cycle, the two parts investigated in this study.

First, Kraft and Hill (2020) studied the use of the Mathematical Quality of Instruction instrument in teacher-coach reflections/debriefs. Their study, particularly relevant to VTPD, suggested rubrics and classroom videos afford coaches’ and teachers’ opportunities to concentrate on important aspects of practice. Kraft and Hill posited that the resources gauge progress in these categories, with resulting changes in teachers’ classroom practices (e.g., giving mathematically correct explanations and other aspects of “ambitious teaching”).

Second, few research findings exist on how coaches develop their craft and what aids in doing so (Kane & Saclarides, 2022); however, using data collected in a Tennessee statewide coaching program, Stein et al. (2022) teased apart how coaches develop effective practices for lesson planning with teachers over time. The main study using the same data showed improved coaching practice in pre-lesson planning conversations (Russell et al., 2020) and improved teachers’ classroom practices.

METHODS

Participant Recruitment

To recruit participants for this PD program, our project team sought teachers and coaches in the United States who were already using IM in their curriculum. The use of IM ensured argumentation lessons provided by VTPD, which adapted IM lessons with more enhanced CMA content, fit into participants’ school-year curriculum and pacing. Another requirement to participate was the need for the teacher and the coach to commit to attending the workshop and complete all coaching cycles throughout the academic year. Though preferred, the coach did not have to specialize in mathematics. Whereas this paper focuses on a specific coach-teacher dyad (described in detail in the following section), the VTPD team provided PD to three coaches and seven middle-school mathematics teachers during the 1st year of the program in the 2021–2022 academic year.

Case Study Participants: Coach Celia and Teacher Belle

Our team conducted a qualitative descriptive case study (Yin, 2014) of a coach’s practice as part of a larger study of how the VTPD program supports mathematics coaches, teachers, and students to engage in CMA. Coach Celia² was selected as a case because she worked intensively with a teacher to complete all phases of a coaching cycle and across six cycles, as recommended by VTPD. She thus showed high commitment to supporting the teacher to engage in VP, enactment, video analysis, and debrief of lessons for each coaching cycle.

At the time of this study, Celia was employed as an instructional coach by her district, coaching teachers at an intermediate school in a suburban city in the southern United States that served students in Grades 5 and 6. Approximately 48% of the students were from racial minority backgrounds, including those who were Hispanic, Black, Asian, or multiracial. She had 5 years of coaching experience. Her content areas included English language arts (ELA), mathematics, science, and social studies. She had a bachelor’s degree in K–6 education and was a nationally board-certified generalist in early childhood education.

For the 2021–2022 academic year, Celia coached one teacher, Belle, for the VTPD program. Belle held a bachelor’s degree in K–6 education and taught mathematics at the school where Celia worked. In the 2021–2022 academic year, Belle taught sixth-grade mathematics. At the start of the study, Celia had worked for three years and Belle had worked for two years at this school/district. Beyond the coach-teacher relationship, Celia and Belle were friends and noted their friendship was a benefit to their work as a dyad during their participation in this PD project because it helped them engage with constructive feedback as they progressed through the coaching cycles.

³All names of coaches, teachers, and students are pseudonyms.

Data Sources and Analysis

The cycles comprising the data set for this study began in September 2021 and ended in May 2022, yielding a complete data set about VP and debrief meetings. In contrast, the other two coaches during this period worked with three teachers each, but these coach–teacher pairs conducted the cycles only partially due to various school and personal constraints. The resulting data set from their cycles was incomplete, offering limited opportunities for our team to study the practices of these coaches over time. Coach Celia’s work with the teacher provided our research team with the most available data, so this coach was selected as a best case, consistent with recommendations for conducting case study research (Yin, 2014) and prior case study research on coach–teacher discourse (Saclarides, 2022).

To better understand the nature of coach–teacher discourse, this article reports key findings from VP and debrief meetings, “each” for Cycles 2 and 5. These cycles were analyzed because they involved focal lessons designed by VTPD (providing our team with necessary context to understand the coach–teacher discourse), and the 6-month-long interval between the cycles provided a sufficiently long time to detect changes in their discourse.⁴

Meeting transcripts comprised the data set for this study, providing early and late data points respectively about coach–teacher discourse. Transcripts were analyzed to understand the topics of the discourse, which revealed the discursive learning opportunities fostered by the coach as she used the structures and tools designed for these meetings. Topics of discourse are indicators of the quality of discourse (Lefstein et al., 2020; Saclarides, 2022).

One member of our team reviewed each transcript holistically to understand the discussion, segmenting the transcript based on topically related exchanges (e.g., materials used in the lesson, the teacher’s goals or strategies, and students’ ideas). The segmentation was discussed and finalized by consensus with another author (Saclarides, 2022; Sherin & Han, 2004). Subsequently, the two authors coded the transcripts independently based on detailed coding schemes developed separately for VP and debrief meetings (see Appendices A and B). The coding schemes were developed deductively and inductively. The deductive approach drew on prior literature on teacher PD (van Es & Sherin, 2010), coaching (Saclarides, 2022), and video-based reflection (Borowiec et al., 2022) to capture various topics (e.g., lesson content, students’ disciplinary thinking, and pedagogical moves) to promote students’ thinking. Topics emerging inductively included lesson setup and discourse moves to provide opportunities for every student to contribute to classroom discourse.

The coding schemes were consistent with the tools and structures designed for VP and debrief conversations, which emphasized discussing (a) concepts and practices while doing the mathematics of a lesson together, (b) teacher moves for access and orchestration to help every student participate, (c) student thinking about mathematical concepts and practices, and (d) cognitive demand of teacher moves. The two authors

first coded and discussed a pair of VP and debrief transcripts. Following this discussion, the authors coded a second pair of VP and debrief transcripts independently and compared their codes to establish interrater agreement. There was 80% agreement on the coded VP data and 87.5% agreement on the coded debrief data. Any discrepancies in coding were resolved through discussion until there was 100% agreement.

After coding each transcript, the number of segments per code was counted to report frequencies of the topics discussed during a cycle. To compare within and between cycles, the frequencies were divided by the total number of segments in the transcript to calculate percentages of the discussion devoted to various topics. The percentages do not add up to 100 because segments were assigned multiple codes when appropriate. Finally, we developed and revised a case study report after receiving feedback from the coach, a member-check measure important for establishing credibility of the findings and interpretations (Guba, 1981).

FINDINGS

The findings highlight various topics that emerged during the coach–teacher discourse, showing how the structures and tools designed by the VTPD program were taken up as the coach and teacher engaged collaboratively with VP and debrief of lessons during Cycles 2 and 5. They used the protocol designed for VP and the discussion guide designed for debrief. The findings about Cycle 2 meetings, followed by Cycle 5 meetings, provide a detailed narrative of the discourse from each meeting, including relevant transcript excerpts to illustrate the main points.

Cycle 2

The focal lesson for Cycle 2 aimed to help students recognize questions about “how many groups?” and involved division operations using diagrams, multiplication equations, and division equations to represent such questions. The argumentation goal was to help students justify conjectures involving division problems in which the divisor and the quotient were not whole numbers (i.e., using fraction models and equations).

Cycle 2 VP

The coach and teacher devoted approximately 35 minutes to this meeting, mostly about potential teacher moves and/or goals for mathematical practices, namely argumentation (6 times, 43%; see Table 1 [data were calculated based on a total of 14 segments and ordered by frequency]). This finding was consistent with the VP protocol (see Figures 4 and 5), which asked them to consider opportunities in the lesson for taking up mathematical practices. They also discussed fractions as they completed the mathematics of the lesson together (4 times, 29%) and considered potential teacher facilitation of student participation (4 times, 29%). They devoted less time to anticipating students’ ideas about the target concepts and mathematical practices (3 times, 21%) and to anticipating students’ participation in classroom discourse (2 times, 14%).

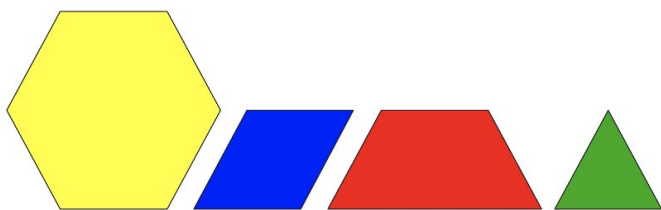
⁴The focal lessons for Cycles 2 and 5 are based on lessons from Illustrative Mathematics (IM, 2021).

Table 1
Percentages of Topics Discussed During Cycle 2 VP Meeting⁵

Topic	f	Discussion (%)
Planning teacher moves/goals for mathematical practices	6	43
Mathematical concepts of a lesson	4	29
Planning teacher facilitation of student participation	4	29
Anticipating student thinking about mathematical concepts	3	21
Anticipating student thinking about mathematical practices	3	21
Anticipating student participation in discourse	2	14
Planning teacher moves/goals for mathematical concepts	2	14
Lesson setup	2	14
Logistics	2	14
Mathematical practices of a lesson	1	7

The following excerpts illustrate how Celia and Belle considered potential teacher moves and/or goals to promote argumentation, which was the most prevalent topic of their discourse. Celia first reviewed a problem on fractions for different shapes, engaging the teacher in doing mathematics together, thereby laying a foundation to visualize and plan for specific pedagogical aspects of the lesson. Celia elicited Belle's thoughts about possible student understandings of the mathematics task and students' interpretations of the justifications presented in the task (see Figure 7). This flow of discourse was consistent with the VP protocol.

Figure 7
Diagram Showing Shapes the Students Discuss in This Lesson



Teacher: I just remember how hard this is for them to understand.
 Coach: What makes it hard?
 Teacher: Because a lot of them will agree with Diego [character in the mathematical problem].
 Coach: Because why?
 Teacher: [Referring to the uncovered part of the trapezoid] This is a third, and that's what's leftover.
 Coach: So, when they say one and a third, what is it that they're considering the whole?
 Teacher: There they're doing this as like an addition

problem. Does that make sense? . . . they think . . . rhombus plus a triangle gets me the trapezoid. They're not thinking rhombus plus what part of rhombus equals a trapezoid?

Coach: And what are they thinking?

Teacher: But instead, a lot of them will see . . . the missing piece is a triangle. So, rhombus plus triangle gets me trapezoid. Well, that's one rhombus plus a third . . . they mix multiplication [and] addition instead.

Finally, to enable Belle to consider potential goals and moves in helping students construct justifications and conclusions, Celia emphasized selecting and sequencing students to share their thinking. She asked clarifying and probing questions to help the teacher look for specific student reasoning during the whole-class discussion. Celia thus kept the discourse focused on teacher and student moves, consistent with the VP protocol.

Teacher: And so, once all teams [have] shared and kind of come up with a final solution, then I would want sharing to happen. So, then, as a whole group, we come to a conclusion.

Coach: Are you gonna select certain kids or look for certain things?

Teacher: It'd be interesting if I had a team that still supported Diego's.

Coach: What kind of reasoning would you look for? . . . For supporting Jada's thinking?

Teacher: Well, I definitely want to make sure that the people that are supporting it, use the rhombuses correctly. . . . They actually remember that's how many rhombuses are in that trapezoid . . . that whole idea of kind of taking the triangle off the table, if that makes sense. . . . And also, if there are students that come up with a multiplication or division equation, I would definitely want to illustrate that . . . "How did you get that?" "How does that support?" . . . I think one of the things about justification, too, . . . I know some students will choose to have diagrams or drawings, so maybe have . . . a variety of examples as well, like using the shapes, maybe drawing it out, having the equations, maybe if there are people that just have . . . written or verbal explanations.

Cycle 2 Debrief

During the debrief (approximately 24 minutes), there was a sharp focus on student participation. Celia and Belle commented most frequently on the teacher's facilitation of full student participation in the enacted lesson (3 times, 30%), on students' actual participation in the discourse (3 times, 30%) of the enacted lesson, and on students' thinking related to argumentation as observed in the enacted lesson (3 times, 30%). These data are included in Table 2, where calculations were based on 10 segments for topics and ordered by

⁵The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

frequency. About halfway through the meeting, they used the rubric from the discussion guide (see Figure 6) to rate various aspects of students' engagement with mathematical practices and the teacher's facilitation and to consider implications based on overall patterns of classroom discourse.

Table 2

Percentages of Topics Discussed During Cycle 2 Debrief Meeting⁶

Topic	f	Discussion (%)
Reflecting on teacher facilitation of student participation (in enacted lesson)	3	30
Reflecting on student participation in discourse (in enacted lesson)	3	30
Reflecting on student thinking about mathematical practices (in enacted lesson)	3	30
Reflecting on student thinking about mathematical concepts (in enacted lesson)	2	20
Planning teacher facilitation of student participation (for future lessons)	2	20
Anticipating student participation (for future lessons)	2	20
Reflecting on teacher facilitation of mathematical concepts (in enacted lesson)	1	10
Reflecting on teacher facilitation of mathematical practices (in enacted lesson)	1	10
Planning teacher moves/goals for mathematical practices (for future lessons)	1	10
Logistics	1	10

Some similarities and differences existed in the discourse of the two Cycle 2 meetings. The coach and teacher discussed the teacher's facilitation and students' thinking and participation during both meetings. These topics were broadly consistent with the purposes, tools, and structures of those meetings. However, they talked about students' actual ideas about mathematical practices and their actual participation in the debrief meeting rather than in the VP meeting, which was consistent with the debrief being designed to reflect on a lived experience of the enacted lesson and additionally on an annotated video of the lesson. Conversely, during debrief, they spent less time discussing the setup or logistics of future lessons and did not discuss the concepts or practices of the lesson. This finding contrasted with the VP meeting, where the latter topics were more pertinent and for which different tools and structures had been designed.

The coach and the teacher reflected together on moments of the enacted lesson they had previously annotated

independently on video. They focused on how students participated and how the teacher facilitated their participation with the help of specific goals, strategies, and structures (e.g., think-pair-share, individual think time), two of the most prevalent topics of the coach-teacher discourse during this debrief meeting. Celia used the discussion guide to help the teacher reflect on students' participation and on her facilitation of their participation in the enacted lesson (see Figure 6 for the rating scale about orchestration under "Teacher Moves"). In doing so, Celia commented on improvements in the teacher's facilitation (and in students' participation) and on how the teacher had used independent think time intentionally to promote more student-directed talk. As evidenced in the following excerpt, the coach also noted changes in students' inclinations to ask questions of their peers. When the teacher voiced a concern about facilitating participation of all students, hinting at a need to modify teaching practices for future lessons, the coach shifted away from talking about the enacted lesson and toward exploring instructional strategies for subsequent lessons to help the teacher improve her practice at orchestrating classroom discourse. In doing so, Celia invited Belle's perspective on specific matters, proposing a strategy to help students engage in questioning with their peers and prompting Belle to consider how she might group students:

- Coach:* I think students are starting to ask each other questions. . . . Because I think that kids are starting to, maybe we can think through how to . . . I don't know, it's been a natural progression so far, right?
- Teacher:* Yeah, I think it's the part of . . . when kids are off, how do I bring them? I think that's where I've got to grow the most.
- Coach:* Do you think it'd be helpful to have . . . questions for students to ask students to guide that discussion for that partner piece?
- Teacher:* But I think about . . . if I have two kids that are already below grade level, they don't know that they're in lost land any better than they're in the right land? Just to be honest.
- Coach:* Are your partnerships set up right now? In like—
- Teacher:* —intentional ways? No. I usually just partner them with whoever they're sitting by . . . rarely have them set up in a particular way because sometimes when we do partner work, I let them choose. Sometimes I just have them partner [with] the person beside them.
- Coach:* If they were intentional, how would you make them intentional?
- Teacher:* Well, my automatic response to that is to put . . . people that are about on the same level together because . . . I have some kids in this class that are . . . seventh-, eighth-grade math ability, I wouldn't put them with like any of my low kids because that's, the low kids will just give in to whatever they say. And so . . . it goes back to the discussions we've had in [professional learning community], where it's,

⁶The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

like, do we put like a 3 and a 5 together, and a 2 and a 4 together? But then . . . I always have this feeling that the upper kid will dominate. And be like, "No, that's dumb." Or like, "You're way off."

Coach: Yeah. So, . . . similar ability.

Cycle 5

The focal lesson for Cycle 5 aimed to help students understand that the surface area of a three-dimensional object with flat faces is a measurement of the combined area of all its faces. The argumentation goal of the lesson was to help students justify the generalizability of their methods for calculating surface area.

Cycle 5 VP

Approximately 36 minutes long, most of this VP meeting was about lesson setup, with Celia and Belle reviewing the lesson tasks, goals, and context (11 times, 74%). They did consider potential teacher moves and/or goals for supporting students' learning of mathematical concepts and practices (5 times, 33% each for concepts and practices), attending to some of the fine-grained aspects of mathematics teaching. Lesser time was devoted to anticipating student thinking. Table 3 provides data calculated based on 15 total segments and ordered by frequency.

Table 3

Percentages of Topics Discussed During Cycle 5 VP Meeting⁷

Topic	f	Discussion (%)
Lesson setup	11	74
Planning teacher moves/goals for mathematical concepts	5	33
Planning teacher moves/goals for mathematical practices	5	33
Anticipating student thinking about mathematical concepts	2	13
Anticipating student thinking about mathematical practices	2	13
Mathematical concepts of a lesson	2	13
Planning teacher facilitation of student participation	1	6

The following excerpt shows how Celia helped Belle consider her goals and moves for facilitating students' learning of mathematical concepts and of argumentation, two of the more prevalent topics during this meeting. After reviewing the lesson goals, mathematics problems, and content, they reviewed a math problem consisting of three parts. Part 1 asked students to find the surface area in square units of a rectangular prism that was 3 units high, 2 units wide, and 5 units long. Part 2 asked students to justify their method, and Part 3 asked students to state whether their method would work for any right rectangular prism. The following excerpt begins where Belle imagined engaging students in partner talk. Celia probed Belle to visualize in greater detail the

student ideas to which she might listen, encouraging Belle's choice of questions to ask during partner talk. In doing so, the coach helped the teacher not only consider her own moves but also attend to potential student thinking related to math practices, thus maintaining a dual focus on both pedagogy and student learning while planning toward curricular content, goals, and tasks:

Teacher: And then maybe we check in they could do partner work or a team check-in, of, what you get and . . . how'd you get there. . . . So, maybe with a partner, "What was your strategy? . . . What's your justification?" I think that I am . . . just kind of observing and just seeing . . . what they're getting.

Coach: Yeah, I like that idea of having those explicit questions out there too, though.

Teacher: And then after that, I don't know if we should have . . . everyone share: "Did we all get the same thing?" to make sure everyone got the same surface area.

Coach: What are you listening in for?

Teacher: Do they get the same answer? And do they have the same justification? Or justification that matches their answer?

Coach: So, while you're . . . actively monitoring, that's when you're listening in to see if they have the same answer.

Teacher: Slash [meaning "or"] correct answer, I guess.

Coach: Okay, and then you're also looking for their method of justifying. What . . . methods of justification do you think you'll see?

Teacher: So, I could see some people . . . counting the faces [of the prism]. So, . . . tactile, using the cube.

Belle considered how some students may count individual squares presented on the faces of the prism. Building on Belle's ideas, Celia helped her visualize how she could elicit students' ideas, as evidenced in the following excerpt:

Coach: So, after . . . you've given [students] the exit ticket, they worked on [Problem Parts] 1 and 2. They thought independently, they pair, then they go back independently to think through [Problem Part] 3. And to decide whether or not the strategy that they're using—the justification—will lead to a generalization where they could use it for any given rectangular prism. And then . . . when you say, "This is . . . what we are sharing," . . . how does that get shared?

Teacher: Well, . . . I guess we could do . . . a pair-share again. And then I think that it would be worth having a couple people share . . . whole group . . . patterns or generalizations.

⁷The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

Cycle 5 Debrief

The debrief meeting lasted approximately 21 minutes, during which Celia and Belle focused on mathematics argumentation and student participation. They commented most frequently on students' thinking related to mathematical practices in the enacted lesson (6 times, 43%), followed by commentary on Belle's facilitation of student participation (5 times, 36%). To a lesser extent, they discussed students' thinking about mathematical concepts (3 times, 21%) and how Belle supported students' understanding of mathematical practices (3 times, 21%). Celia helped Belle take stock of how students' learning and her facilitation were progressing toward her goal of participation by all students in CMA. In fact, in contrast to the Cycle 2 debrief meeting, the discussion guide was used early in this meeting after the coach had finished reviewing the lesson goals and the teacher's pedagogical goals and moves. Table 4 provides data calculated based on 14 segments for topics ordered by frequency.

Table 4
Percentages of Topics Discussed During Cycle 5 Debrief Meeting⁸

Topic	f	Discussion (%)
Reflecting on student thinking about mathematical practices (in enacted lesson)	6	43
Reflecting on teacher facilitation of student participation (in enacted lesson)	5	36
Reflecting on student thinking about mathematical concepts (in enacted lesson)	3	21
Reflecting on teacher moves/goals for mathematical practices (in enacted lesson)	3	21
Planning teacher facilitation of student participation (for future lesson)	3	21
Reflecting on teacher goals/moves for mathematical concepts (in enacted lesson)	2	14
Reflecting on student participation in discourse (in enacted lesson)	2	14
Revisiting curricular tasks/goals (in enacted lesson)	1	7

The following excerpt, in which the coach commented on Belle's ratings of her students' engagement with conjectures and justifications, demonstrates how Celia used the discussion guide to structure Belle's reflection on students' thinking about argumentation and her facilitation of student participation, which were among the more prevalent topics of the coach-teacher discourse (see Figure 6). The discussion guide helped them to better understand the different levels of justification (Knuth et al., 2009). Building on Belle's comments, Celia shared her own observations and interpretations about students' thinking, relating these back

to what they had anticipated previously, to help Belle assess her students' progress toward mathematical argumentation. In doing so, Celia added details about students' ideas and reasoning she had noted independently in the video of the enacted lesson:

- Coach:** *I thought Peter [a student] did an amazing job of communicating his thinking with how to do that. And . . . I was a little bit nervous when they started talking about length times width times height because I knew that volume was one of those areas that we had anticipated that they might have some misconceptions with. So, I thought it was really cool when they were able to . . . Leigh [a student] that was able to bring that up and make that connection at the end of like, how you could use the length and the height.*
- Teacher:** *So, I don't know if we want to do a 3 on that one [referring to rating students' conjecturing on the rating scale provided in the discussion guide]. And it's more of like, maybe like a 2.5. Okay. And then as far as justifying [referring to student moves for justification described in the discussion guide]. There's Level 1, Level 2, and Level 3.*
- Coach:** *I mean, I think for the kids that came up, they were able to . . . apply this to a general thing in order to, essentially, . . . put into words what the formula was before writing the formula.*

As shown in the following excerpt, the coach continued to share her observations and inferences about specific teacher moves, focusing on her orchestration of classroom discourse and her facilitation of student access, which were criteria highlighted in the discussion guide. In doing so, the coach posed questions to help the teacher ponder next steps, and she tailored suggestions to areas of improvement identified by the teacher:

- Teacher:** *And then, access [referring to the rating scale in the discussion guide].*
- Coach:** *I think that think-pair-share really helped with the access.*
- Teacher:** *And allowing them to have independent think time to . . . I know, for some of my . . . quieter students, or slower processors, whatever. They have some time to think, like, "Does this work?" And I was pretty impressed that most of them were able to put something on the paper. Because sometimes I feel like students will just sit there—*
- Coach:** *—and wait.*
- Teacher:** *Or they'll . . . immediately be, like, "I don't know what I'm doing." I think I see . . . that pattern more in my first class because they just don't feel so successful in math. And so, it*

⁸The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

was really exciting to see at least they were all trying, and . . . attempting something. So, let's see—don't necessarily feel like I called anyone specific that hadn't participated.

Coach: *How do you push for more either?*

Teacher: *It's hard for me because . . . in my mind, I'm like, "Oh, we'll just ask the kids that aren't sharing anything." But then . . . maybe they don't have anything to share.*

Coach: *So, when you're doing that monitoring piece, maybe you start by monitoring those kids that are typically required . . . to see if there's something they could do. For yeah, like start with them as . . . what you're looking for.*

The findings show the broad goals, setup, and mathematical concepts and practices of a lesson were more prevalent topics during VP than during debrief in Cycle 5. These topics were aligned with the purpose and support designed for VP. Furthermore, there was greater discourse about students' actual mathematical thinking and participation and about the teacher's actual facilitation of argumentation and participation of all students during debrief than during VP, as the coach helped the teacher use the discussion guide rating scales to consider students' progress in constructing arguments and the teachers' moves in orchestrating discourse and providing access for participation. Finally, the coach and teacher devoted more time to planning facilitation of participation of all students for a future lesson during debrief than during VP in Cycle 5, for which the rating scales about teacher moves (i.e., orchestration and access) in the discussion guide provided a clear basis to formulate goals and next steps.

Comparing Coaching Cycles 2 and 5

Across Cycles 2 and 5, the coach supported the teacher to attend to both mathematical argumentation and participation of all students. Celia helped Belle consider not only her goals and moves for mathematical argumentation and student participation but also the progress her students made in terms of their mathematical thinking and participation in argumentation. Celia drew on the meeting protocol designed by the VTPD program. She asked clarifying and probing questions and responded with suggestions, observations, and interpretations to engage the teacher in (a) doing mathematics together; (b) anticipating her students' ideas, experiences, and challenges; (c) envisioning teacher moves; and (d) reflecting on how teacher facilitation, student learning, and participation unfolded during an enacted lesson.

There were also key differences in the topics between the cycles (see Tables 5 and 6). Cycle 5 VP focused more on broad lesson setup and less on the fine-grained aspects

of their planning compared to Cycle 2 VP. Furthermore, compared to Cycle 2 VP, there was less time devoted in Cycle 5 VP to doing mathematics together (i.e., to working out the mathematical concepts and practices of the lesson). Finally, compared to Cycle 2 debrief, there was greater talk in Cycle 5 debrief about (a) how students were thinking, especially about mathematical practices of the enacted lesson; and (b) the teacher's moves and goals in facilitating mathematical concepts, practices, and participation of all students.

Table 5

Comparison of VP Meeting Topics During Cycles 2 and 5⁹

Topic	Discussion during Cycle 2 VP (%)	Discussion during Cycle 5 VP (%)
Planning teacher moves/goals for mathematical practices	43	33
Planning teacher moves/goals for mathematical concepts	14	33
Planning teacher facilitation of student participation	29	6
Discussing mathematical concepts of a lesson	29	13
Discussing mathematical practices of a lesson	7	0
Anticipating student thinking about mathematical concepts	21	13
Anticipating student thinking about mathematical practices	21	13
Anticipating student participation in discourse	14	0
Lesson setup	14	74
Logistics	14	0

⁹The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

Table 6*Comparison of Debrief Meeting Topics During Cycles 2 and 5¹⁰*

Topic	Discussion during Cycle 2 debrief (%)	Discussion during Cycle 5 debrief (%)
Reflecting on teacher facilitation of student participation in CMA (in enacted lesson)	30	36
Reflecting on teacher moves/goals for mathematical concepts (in enacted lesson)	0	14
Reflecting on teacher moves/goals for mathematical practices (in enacted lesson)	0	21
Reflecting on student thinking about mathematical practices (in enacted lesson)	30	43
Reflecting on student thinking about mathematical concepts (in enacted lesson)	20	21
Reflecting on student participation in discourse (in enacted lesson)	30	14
Anticipating student participation (for future lessons)	20	0
Planning teacher facilitation of student participation (for future lessons)	20	21
Planning teacher moves/goals for mathematical practices (for future lessons)	10	0

DISCUSSION

This study revealed how PD structures and tools designed for lesson visualization, planning, and debriefing inform the topics of a coach's discourse with a teacher in promoting CMA for all students. The topics of coach-teacher discourse shifted within and between phases of Cycles 2 and 5. The differential use of the discussion guide may explain the differences observed between debrief meetings of Cycles 2 and 5 with respect to the extent to which the discussion was devoted to students' thinking and the teacher's moves and goals in facilitating the lesson. The rubrics focused and structured reflection on how students were conjecturing and justifying their mathematical arguments and on how the teacher worked toward cognitively demanding questions, orchestration, and access in classroom discourse. These rubrics were used early in Cycle 5 debrief to structure coach-teacher reflection, whereas the rubrics were introduced after almost half the meeting in Cycle 2 debrief.

On the other hand, the VP meetings did not show this kind of growth over time (except planning for the teacher's facilitation of mathematical concepts, which was more frequent in Cycle 5 than Cycle 2). A possible reason for this result concerns differences in the focal lessons for the two cycles. The Cycle 5 lesson about finding the surface area of a rectangular prism involved a three-part mathematical argumentation task for which students needed to calculate the surface area and justify their method and its generalizability for any rectangular prism. In contrast, the Cycle 2 lesson was more straightforward because it did not require students to justify the generalizability of their reasoning. A more complex, multistep mathematical argumentation task in Cycle 5 may have led the coach and the teacher to devote more time to reviewing the setup of the lesson than in Cycle 2.

The VP phase provided a dedicated time for the coach and teacher to brainstorm alternative paths through which a mathematical argumentation task could unfold. This phase included templates to help the coach and teacher attend to mathematical concepts and practices, pedagogical goals, and teacher and student moves with an overarching emphasis on CMA for all students. Additionally, principles of colearning and collaboration undergird many content-focused coaching models (Campbell & Griffin, 2017). The VP phase offered a structured opportunity for the coach and teacher to do mathematics together in the context of specific lessons, working as equals to make sense of the lesson content and to generate and reinforce shared goals for teaching and learning.

There is a lack of research focused on the reflection phase of the coaching cycle (Saclarides, 2022). In the VTPD program, the reflection phase is broken into two parts: (a) analyzing and commenting on the video independently and (b) subsequently a debriefing based in part on the video analysis. The coach drew on a clear structure and tools provided by the PD to engage the teacher with various topics based on her own independent viewing of the classroom video and emergent interactions with the teacher. Saclarides's (2022)

¹⁰The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

case study portrayed reflection sessions that went deep into student thinking and teaching moves, as did this case study. Saclarides noted not all coaches are as expert as the coach in her case study and, therefore, may need further opportunities for developing coaching skills. This PD program provided an example of what that support could look like.

Furthermore, the coach's uptake of the discussion guide for the debriefing was important because it drew the discussion to several issues. The coach and the teacher noted the cognitive demand of the teacher's questioning, students' engagement with the conjectures and justifications, and the teacher's orchestration of classroom discourse and provision of access to CMA. With these rubrics, the coach guided the teacher to not only consider her own facilitation but also to consider students' thinking and participation in important disciplinary practices. When discussing access, the coach and teacher noticed how more students were drawn into class participation, attributing this shift to moves the teacher had made using the discussion guide's term "invitations" to a broad group of students. Similarly, when discussing cognitive demand, the coach and the teacher focused on student participation when they noted how the teacher's use of open-ended questions led a student who "was often quiet" to make a substantial contribution to class discussion.

Across the two cycles, there was generally greater attention to students' thinking and participation during debrief than during VP. This finding was expected due to differences in the purposes and support designed for VP and debrief meetings. Debrief meetings centered on reflecting on the past, with classroom events that could be reviewed on video and were addressed in the discussion guide, whereas VP meetings focused on envisioning student actions. The discussion guide also served as a tool for the coach and teacher to explore next steps and set goals jointly, grounding their work in the lived experience of the classroom and reinforcing a mutual, collaborative relationship between coach and teacher.

Finally, in some mathematics coaching programs, the coaches select video segments and comment on the segments before the teacher does (Gregory et al., 2017). In the VTPD program, coaches and teachers comment on the video concurrently and separately, made possible by cloud-based technologies. This approach to using video may add to the collaborative nature of coach-teacher interactions.

Limitations of the Study

This study focused on the case of a coach's practice for supporting a teacher by taking up PD structures and tools designed for visualizing, planning, and debriefing of lessons. However, the findings do not reveal how a coach may support multiple teachers and how a coach's practice may shift during subsequent rounds of implementing the coaching cycles. Furthermore, whereas the study shone light on the discursive learning opportunities fostered by the coach, it did not analyze other important indicators of the quality of coach-

teacher discourse, such as its depth or the extent to which the discourse includes reasoning, evidence, or details to substantiate claims (Lefstein et al., 2020; Saclarides, 2022).

Recommendations for Research on PD for Coaches

This study focused on the coaching practice of a single coach working with a teacher during a year of participation in a PD program. Future researchers may examine shifts in a coach's practice over longer term participation in PD and in response to multiple teachers. For instance, what levels of depth are attained while reflecting collaboratively on students' thinking and participation? How does a coach tailor support in response to teachers' own understandings of mathematical practices and in response to their understandings of students' prior experiences with CMA? Furthermore, cross-case analyses may compare practices of other coaches in supporting CMA for all students.

Recommendations for Designing Coaching Programs

This study leads to a number of recommendations to help other designers in developing PD programs that include coaching. First, we recommend use of well-structured protocols and routines to guide coach-teacher dyads in productive discourse. Protocols and routines are foundational to current coaching designs (Gibbons & Okun, 2023; Kraft & Hill, 2020; Stein et al., 2022). At the broadest level, a handful of structures—all of which can be introduced in joint coach-teacher PD or in coach-only PD—are useful in the coach-teacher dyad to further productive discourse. However, there is a balance to strike between structures that may be overly constrictive and structures that may be too open to provide adequate support for productive discourse. In the VTPD program, the balance was tilted toward more highly structured tools and routines that still enabled flexible use. This tilt was evidenced by the small list of topics the dyad spent most of their time discussing during VP and debrief meetings.

VP is a core set of routines that can be adopted or adapted in other programs. Excerpts from Cycle 5 showed the routine is flexible in supporting teachers' top-of-mind concerns. We recommend PD designers consider the phases through which teachers may move in the routines they design to ensure the routines are flexible enough to support the coach-teacher dyad through those phases, which may not be where designers imagined they would begin.

The debrief phase of VTPD coaching cycle relies on a discussion guide that presents a set of relevant, research-based categories to help situate teachers' practice in that research. Such research-based categories can ground discussions of practice that not only analyze enactments of prior lessons but also can lead to consideration of future improvements in practice. Therefore, PD designers may wish to similarly distill and efficiently present research-based categories relevant to their aims.

CONCLUSION

Professional organizations like the National Council of Supervisors of Mathematics (2024) have long envisioned that coaches and emerging school leaders need the “understanding, knowledge and skills to continue their own personal growth . . . and further develop excellence in mathematics teaching” (para. 3). This article reported on a PD program that includes coaching, combining aspects of the familiar coaching cycle and some innovative elements to support (a) the coach and teacher to engage in colearning during the summer institute and do the mathematics together during year-round coaching cycles, (b) the process of VP to anticipate students’ thinking and teacher moves, (c) both coach and teacher independently reflecting on and annotating video of enacted lessons prior to joint debriefing, and (d) enacted lessons based on research-based criteria characterizing classroom discussions. The study also exemplifies how structures and tools of the PD program designed for VP and debrief inform the topics of a coach’s discourse with a teacher in promoting CMA for all students. This study’s recommendations are applicable to the work of other PD designers, researchers, and practitioners in this area, in the same way prior research has informed VTPD.

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Appendix A: Visualization Planning Coding Scheme

Main code and definition	Subcodes	Definitions of subcodes
Math content: Coach and teacher make sense of the lesson content as they do the math together.	Math content – concepts	The discussion focuses on math concepts targeted in the lesson task.
	Math content – practices	The discussion focuses on the math practices targeted in the lesson task. Indicate the practice while applying this code.
Planning teacher moves/goals: Fine-grained decisions and consideration of potential moves and personal pedagogical goals for supporting students' learning. The moves may go beyond verbal discourse (e.g., questions or statements a teacher might make) to include provision of specific resources.	Planning teacher moves/goals – math concepts	The discussion focuses on teacher moves/goals related to specific math concepts targeted in the lesson tasks.
	Planning teacher moves/goals – math practices	The discussion focuses on teacher moves/goals related to specific math practices targeted in the lesson tasks. Indicate the practice while applying this code.
Anticipating student thinking/moves about mathematical concepts and practices: Anticipate student ideas and reasoning, prior knowledge and experiences, potential challenges, or misconceptions about math content. Imagine student actions and responses, as recommended in the Visualization Planning template.	Anticipating student thinking/moves – math concepts	The discussion focuses on potential student actions and responses related to specific math concepts targeted in the lesson tasks.
	Anticipating student thinking/moves – math practices	The discussion focuses on potential student actions and responses related to specific math practices targeted in the lesson tasks. Indicate the practice while applying this code.
Planning for participation of all students: Consider opportunities, challenges, prior experiences with orchestration, and access for student participation in math discourse. These factors may include commentary about participation structures (e.g., small group, whole-class, partner work).	Planning for participation – teacher facilitation	The discussion focuses on teachers' prior experiences, anticipating their actions, personal pedagogical goals, concerns, challenges in orchestrating classroom discourse, and providing students with access to the discourse.
	Anticipating student participation	The discussion focuses on students' prior experiences, anticipating their actions, and challenges with participation in and access to classroom discourse.

Appendix B: Debrief Coding Scheme

Code	Definition
Student thinking/moves – math concepts [enacted lesson]	Discuss students’ emergent understanding, difficulties, preconceptions, etc., about math concepts.
Student thinking/moves – math practices [enacted lesson]	Discuss students’ emergent understanding, difficulties, preconceptions, etc., about math practices emphasized in the VTPD program.
Teacher moves/goals – math concepts [enacted lesson]	Discuss the teacher’s actions/goals for supporting students’ understanding of mathematical concepts. The pedagogical goals reflect the teacher’s personal intentions for supporting students’ learning, which may account for students’ specific needs, interests, etc., in contrast to more general curricular or lesson goals.
Teacher moves/goals – math practices [enacted lesson]	Discuss the teacher’s actions /goals for supporting students’ understanding of mathematical practices. The pedagogical goals reflect the teacher’s personal intentions for supporting students’ learning, which may account for students’ specific needs, interests, etc., in contrast to more general curricular/lesson goals. Commentary may refer to the construct of “cognitive demand,” emphasized in the Discussion Guide.
Student participation [enacted lesson]	Discuss opportunities, challenges, changes, or patterns observed in students’ participation in and access to classroom discussions. Commentary may refer to orchestration and/or access, as recommended in the Discussion Guide.
Teacher facilitation of student participation [enacted lesson]	Discuss opportunities, challenges, changes, or patterns observed in teacher’s orchestration and provision of access to classroom discussions.
Revisiting curricular tasks/goals	Discuss general lesson tasks/goals. These may be documented in the lesson plan.
Logistics	Discuss technology for videorecording/video analysis, coaching cycle schedule, etc.
Planning teacher moves/goals – math practices [for future lesson]	Discuss potential strategies or goals for supporting students’ understanding of math practices for subsequent lessons.
Anticipating student participation [for future lesson]	Discuss potential opportunities, challenges, needs, etc. about students’ participation for subsequent lessons.

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