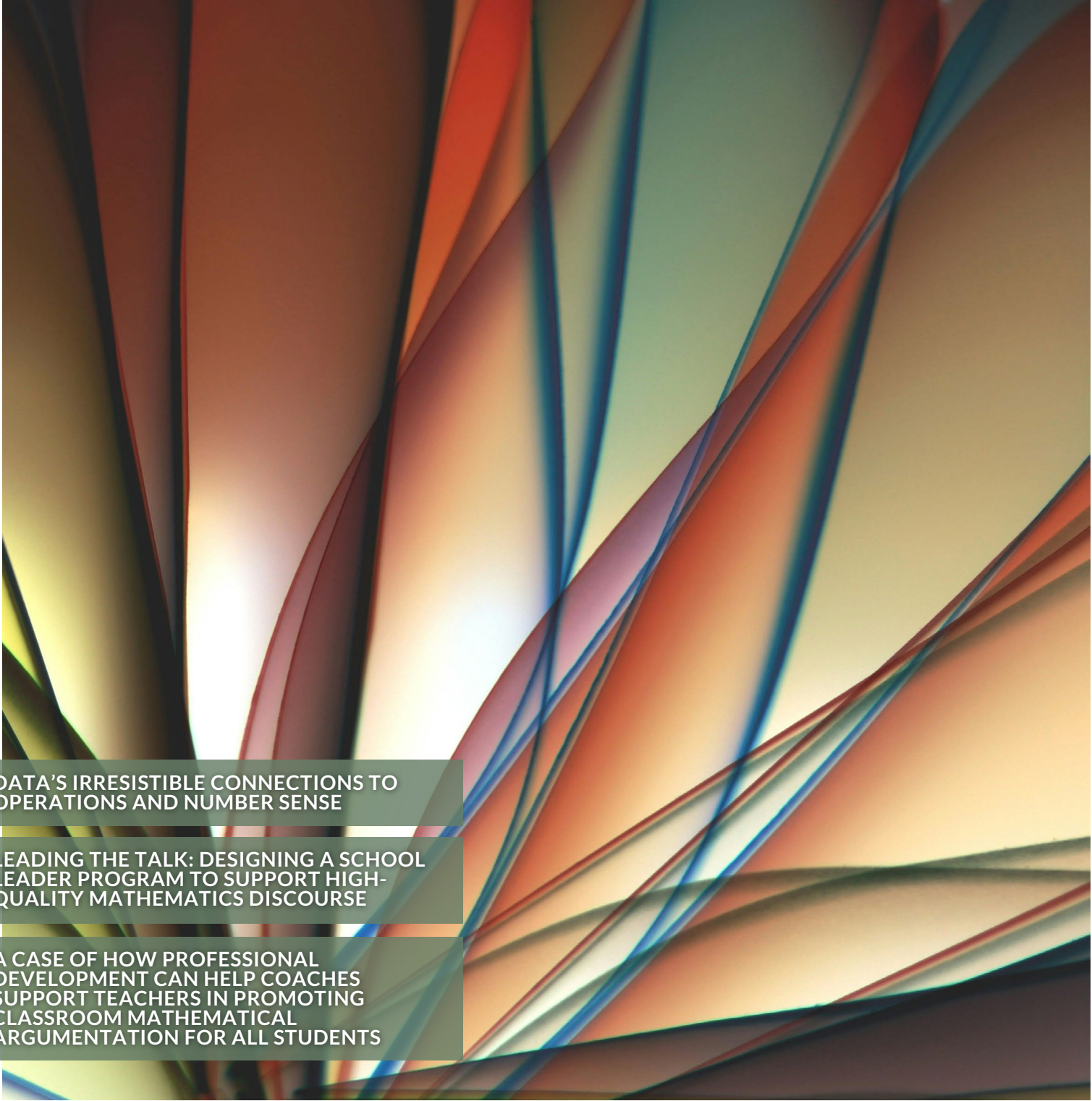


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OPERATIONS AND NUMBER SENSE

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LEADER PROGRAM TO SUPPORT HIGH-
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This article discusses a workshop designed to enhance elementary teachers' understanding of how data, number sense, and operations are interconnected. Despite the important role of data in the Common Core State Standards, many teachers have reported feeling unprepared to teach data concepts. Accordingly, the workshop engaged teachers in hands-on activities (i.e., data blocks, datafied counting collections, graph-talk routines) designed to deepen their knowledge and confidence in teaching data. These activities helped teachers explore connections between data and other math concepts (e.g., comparison, counting, estimation). The article highlights the importance of viewing mathematics as an interconnected whole and offers strategies to integrate data-rich activities into classroom instruction, promoting deeper mathematical understanding and student engagement.

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This article presents the design and evaluation of the Project All Included in Mathematics (Project AIM) School Leader Program (SLP), which supports principals in fostering high-quality mathematics discourse. We used design research to conduct three studies examining (a) SLP's design principles, (b) activity alignment with the National Council of Supervisors of Mathematics (NCSM) essential actions (EA) framework for leadership in mathematics education, and (c) principals' perspectives on SLP's contribution to their learning and leadership practices. Findings showed strong alignment between principals' needs and the design principles, consistent connections between SLP activities and EA, and key principals' perspectives, highlighting growth in instructional vision, capacity, and instructional support. Results also underscore the importance of aligning leader and teacher learning, embedding practical leadership tools, and approaching EAs as a guide when designing professional development (PD) for school leaders.

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by Anushree Bopardikar, Jennifer Knudsen, Christina B. Silva, Harriette S. Stevens

This article overviews how professional development (PD) can help coaches support teachers in fostering classroom mathematical argumentation (CMA) for all students. After summarizing conceptual underpinnings of a PD program on coaching, the article describes the program, presenting a case study of how structures and tools designed by the PD for lesson planning and reflection informed a coach's discourse with a teacher. Analyses of transcripts showed the extent to which a coach-teacher dyad discussed various topics related to mathematical concepts and practices and students' participation during early and late coaching cycles, informed by the purposes and protocols designed to visualize, plan, and debrief lesson enactment. The article highlights recommendations for PD design and research to support mathematics coaches.

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ABSTRACT

This article presents the design and evaluation of the Project All Included in Mathematics (Project AIM) School Leader Program (SLP), which supports principals in fostering high-quality mathematics discourse. We used design research to conduct three studies examining (a) SLP's design principles, (b) activity alignment with the National Council of Supervisors of Mathematics (NCSM) essential actions (EA) framework for leadership in mathematics education, and (c) principals' perspectives on SLP's contribution to their learning and leadership practices. Findings showed strong alignment between principals' needs and the design principles, consistent connections between SLP activities and EA, and key principals' perspectives, highlighting growth in instructional vision, capacity, and instructional support. Results also underscore the importance of aligning leader and teacher learning, embedding practical leadership tools, and approaching EAs as a guide when designing professional development (PD) for school leaders.

Research has highlighted the important role principals play in the success of their schools, teachers, and students. Principals' support for teachers' professional learning enables teachers to adapt continually to the complexities of teaching mathematics (Nelson, 1998). Strong principal support has been linked to improved teacher attitudes, preparedness for instruction, and implementation of effective classroom practices (Heck et al., 2008). Principals account for approximately one quarter of school effects (i.e., second only to teachers) on student outcomes (Leithwood et al., 2004), making principals and their work a key phenomenon of interest for student success.

Although studies on the importance of principals as instructional leaders have gained attention in the education leadership literature, relatively few scholars have focused on how to support principals as they develop the knowledge and practices needed to promote teachers' professional growth and instructional improvement. More

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specifically, in mathematics, despite the well-documented importance of principal practices for promoting high-quality mathematics instruction (HQMI), professional development (PD) efforts for principals in the context of mathematics reform initiatives have often focused narrowly on developing principals' content knowledge (Boston et al., 2017; Steele et al., 2015) or on classroom management and organizational strategies (Cunningham & Lochmiller, 2020). The lack of coherent systems connecting leadership roles (e.g., principals) with teacher learning poses a significant barrier to sustained instructional improvement in mathematics (Kazemi & Resnick, 2020). Therefore, it is important to design PD that build principals' capacity to foster instructional practices that support HQMI, aligning leadership development with teacher learning.

In this paper, our research team shares a set of design principles and activities used in a PD program for school leaders interested in fostering HQMI. We contend these tested principles and types of activities can support other school leaders interested in augmenting their knowledge of how to support quality instruction for teachers and learners at their own schools. As part of a larger design research study, we developed the School Leader Program (SLP) as one component of the larger initiative called Project All Included in Mathematics (Project AIM). In what follows, we first briefly review the literature on principals' knowledge for promoting HQMI. Next, we provide information about Project AIM, the context for the AIM-SLP, and its focus on high-quality mathematics discourse (i.e., a key feature of HQMI). Then, we share the design principles and related activities created for the AIM-SLP. Finally, we present the results of three short studies used to test our design. These studies examined the following questions:

- In what ways do the PD principles derived from the literature align with what school leaders consider the knowledge, skills, and resources needed to implement high-quality mathematics discourse?
- In what ways do the designed AIM-SLP activities align with the essential actions (EA) framework for leadership in mathematics education outlined by the National Council of Supervisors of Mathematics (NCSM)?
- What are the perspectives of participating principals regarding the contributions of the AIM-SLP to their support for high-quality mathematics discourse?

Principals' Knowledge and Practices for Promoting HQMI
In mathematics, principals play a crucial role as school instructional leaders by establishing instructional vision, ensuring teachers have necessary resources and professional

learning opportunities, and creating conditions that enable teachers to deliver HQMI (Boston et al., 2017; Cunningham & Lochmiller, 2020). Supporting HQMI necessitates that principals cultivate a shared vision of teaching and learning rooted in meaningful and engaging mathematics learning experiences for all students (Cobb et al., 2018; Munter, 2014). Effective leadership practices that support instructional improvement in mathematics include fostering teacher collaboration through professional learning communities, conducting classroom observations, providing constructive feedback, and building professional capacity (Goddard et al., 2015; Hitt & Tucker, 2016; Kazemi & Resnick, 2020). Principals who prioritize these leadership activities tend to cultivate positive school cultures that promote academic success for all students (Sebastian et al., 2018; The Wallace Foundation, 2013).

Supporting HQMI also involves developing a shared vision of teaching and learning rooted in meaningful and engaging mathematics learning experiences for all students (Cobb et al., 2018; Munter, 2014). Research has suggested principals need mathematical and pedagogical content knowledge to engage meaningfully in instructional discussions with teachers and support instructional practices that promote student learning and engagement (Corcoran, 2017; Cunningham & Lochmiller, 2020). This expectation presents a challenge for principals who must support high-quality instruction in all content areas. In fact, some researchers have considered whether principals can realistically become content specialists for every subject area (Fink & Resnick, 2001).

Stein and Nelson (2003) addressed this tension through the concept of leadership content knowledge, which proposes that “principals need an enhanced understanding of effective teaching and learning practices in specific content areas” (Boston et al., 2017, p. 184) rather than deep subject expertise. Boston et al. (2017) also stated, “Principals need sufficient knowledge of what constitutes HQMI” (p. 184), which can help them effectively conduct classroom observations, provide helpful feedback to teachers, and establish a clear vision with high expectations for mathematics instruction (Kazemi & Resnick, 2020).

One effective strategy for principals to enhance their understanding of HQMI and become attuned to teachers’ needs is to engage as colearners in teacher PD focused on mathematics content and pedagogy. Through active participation in teacher PD sessions, team meetings, and collaborative lesson planning, principals can deepen their understanding and knowledge of effective mathematics instruction and reinforce their roles as instructional leaders (Cunningham & Lochmiller, 2020; Goddard et al., 2015; Grissom et al., 2021; Kazemi et al., 2022). This engagement not only strengthens their capacity to support instructional improvement, but also fosters continuous learning and collaboration within school culture (West, 2017). For many principals, time constraints and competing demands require strategic choices about when to participate in PD designed for their teachers.

Although sufficient knowledge is crucial for instructional leadership, effective principals are also expected to apply this knowledge in specific leadership practices to support HQMI. Several research-documented practices (e.g. Boston et al., 2017; Hitt & Tucker, 2016) can strengthen support for effective instruction. First, beyond establishing a shared instructional vision, principals must align school goals and communicate expectations that reflect a commitment to this vision (Hitt & Tucker, 2016). Integrating this vision with classroom practices requires that principals communicate expectations for HQMI (Boston et al., 2017; Hitt & Tucker, 2016) and enhance their own understanding of HQMI while attending to teachers’ professional learning needs (Kazemi & Resnick, 2020; Nelson, 1998). This dual focus on their own growth and that of their teachers positions principals as collaborative leaders who can connect instructional vision and classroom implementation (Nelson, 1998).

Principals who provide consistent instructional support to teachers and create professional learning opportunities are better positioned for effective implementation of HQMI. This support can include leadership activities like reflecting on lesson plans, conducting classroom observations, providing actionable constructive feedback, and creating reflective spaces for professional growth (Grissom et al., 2021; Hitt & Tucker, 2016; Sebastian et al., 2016). Principals play an important role in addressing contextual challenge to foster a supportive school environment (e.g., limited time, resources, and opportunities for teacher learning; LaVenia, 2015; May & Supovitz, 2011). In addressing these challenges, principals can distribute leadership by working collaboratively with teacher leaders and instructional coaches to build instructional support capacity across the school (Cunningham & Lochmiller, 2020; Hitt & Tucker, 2016). This collaborative leadership approach ensures coaches, teachers, and principals support teachers’ learning collectively and attend to a shared vision and goals for instruction.

Trustful relationships and accountability are fundamental elements to the success of key leadership practices. Trust between principals and teachers fosters open communication, collaboration, and a shared commitment to HQMI and instructional improvement. Principals build trust by providing dependable support, recognizing teachers’ needs, and valuing teachers’ input in decision-making processes (Hitt & Tucker, 2016). A culture of trust encourages teachers to implement newly learned instructional practices, seek feedback, and engage in continuous professional growth, all of which contribute to enhanced student learning outcomes (Hitt & Tucker, 2016). Accountability includes setting measurable goals, monitoring progress, and providing constructive feedback to staff. By growing a culture of responsibility, principals encourage teachers to take ownership of their professional growth and student outcomes (Hitt & Tucker, 2016).

The research on effective instructional leadership practices of (a) establishing a shared vision, (b) providing consistent instructional support, (c) addressing contextual challenges, (d) distributing leadership, (e) cultivating trust, and (f) ensuring accountability informed the creation of design

principles for our AIM-SLP. These practices also aligned with established frameworks in mathematics education leadership such as the NCSM's (2020) EA framework for leadership in mathematics education, which provides structured guidance for translating research-based practices into actionable principles for leaders at all levels. In what follows, we return to the EA framework as a tool to examine AIM-SLP.

CONTEXT: PROJECT AIM

Project AIM is a long-term PD initiative designed to enhance K–2 mathematics instruction. Project AIM focuses on the promotion of high-quality mathematics discourse in the classroom, a key feature of HQMI. In Project AIM, *mathematics discourse* is defined as “patterned ways of using questioning, explaining, listening, and different modes of communication in the classroom to promote conceptual understanding in math for all learners” (Sztajn et al., 2021, p. 7). High-quality mathematics discourse uses different patterns to support all strands of mathematics proficiency (Kilpatrick et al., 2001). This type of discourse “is purposeful and engages students in taking responsibility for their own learning and for the learning of their peers.” (Sztajn et al., 2021, p. 12).

Beginning in 2010, Project AIM received a series of grants from the National Science Foundation to develop and test comprehensive PD materials for K–2 mathematics. These materials, now stored and facilitated through a learning management system, were created for scalable, district-led implementation and include a core teacher PD program (AIM-PD) and facilitator preparation program (AIM-FP). Classroom-, school- or district-level mathematics leaders who participate in the AIM-FP learn about the AIM-PD and receive access to all AIM-PD materials. They can then locally facilitate the 30-hour AIM-PD for teachers, which is designed for synchronous participation of school teams.

Our research team tested and revised the AIM-PD materials across six implementation cycles that involved over 230 teachers and 14 facilitators. Findings from these implementations demonstrated improvement in teachers' mathematical knowledge for teaching, beliefs about importance of student discussion, preparedness to facilitate discourse-rich instruction, frequency of planning for discourse, and use of student discourse methods during instruction (Sztajn, 2021). In Project AIM-PD, teachers engage with a variety of mathematics discourse techniques (Sztajn et al., 2021). The AIM-PD “flipped the script” (Guskey, 2020) on PD design to initiate new teaching practices rapidly. AIM-PD starts with lesson organization, moves quickly into the implementation of purposeful AIM discourse techniques, and circles back to connect teachers' reflections on implementation to their knowledge about high-quality mathematics discourse (Sztajn et al., 2020). Throughout the PD, teachers apply the techniques they learn in their classrooms, use their experiences to refine their implementation, and deepen their understanding of high-quality mathematics discourse and how it enhances student learning.

To further enhance the AIM-PD impact, it is important that teachers transform what they learn in the PD into action. Thus, we designed two additional programs aligned with the overall goals of Project AIM: the AIM Coaching Program (AIM-CP) for instructional leaders and the AIM-SLP for school principals, the latter of which is the focus of this paper.

THE PROJECT AIM-SLP

AIM-PD focuses primarily on classroom discourse, 1 of 3 dimensions of HQMI alongside (a) the nature of mathematical tasks and (b) the role of the teacher (Munter, 2014). The nature of mathematical tasks includes selecting high-quality tasks that challenge students to engage with various problem-solving strategies, connect their ideas to mathematical concepts, use multiple representations, and justify their thinking rather than simply focusing on procedural calculations (Munter, 2014; Stein & Smith, 1998). The role of the teacher extends beyond delivering content to involve the orchestration of classroom activities that promote mathematical exploration and position students as active thinkers and doers of mathematics. Classroom discourse involves creating a learning community where students engage in meaningful mathematical conversations, critique ideas, and justify their reasoning (National Council of Teachers of Mathematics, 2014).

The design of AIM-SLP emphasizes to principals the importance of high-quality discourse to create a learning environment where students actively explore mathematical ideas and take ownership of their learning, supported by teacher facilitation and challenging tasks (Munter, 2014). This type of mathematics discourse is purposeful and inclusive, requiring student interactions and engagement to promote conceptual understanding of mathematics and build procedural fluency and productive dispositions (Sztajn et al., 2021). It involves (but is not limited to) consistent patterns of questioning, explaining, listening, and multimodal communication embedded into daily classroom norms. Teachers play a vital role in fostering these patterns, ensuring all students participate meaningfully in classroom discourse (Sztajn et al., 2021) by listening, justifying, critiquing, and making sense of mathematical ideas (Dillon et al., 2022; National Council of Teachers of Mathematics, 2014; Stein & Smith, 1998).

Recognizing the central role of school leaders in supporting instructional improvement, AIM-SLP places a strong emphasis on principals' understanding of high-quality mathematics discourse as a key factor in driving meaningful change. This emphasis is supported by a growing body of research indicating that principals' conceptions of mathematics instruction directly influence their leadership practices and the instructional quality in their schools (Kazemi et al., 2022). Principals' visions are more predictive of teachers' understanding and enactment of instructional goals than the time principals spend engaging in instructional leadership activities directly (Katterfeld, 2013). Therefore, it is important to support principals in developing a clear understanding of high-quality mathematics discourse for

strengthening coherence between leadership practices and classroom instruction.

Using Design Research in the Development of the AIM-SLP

In Project AIM, we applied educational design research to develop our PD materials, including the AIM-SLP. Educational design research is an iterative process of systematic inquiry for creating and testing effectiveness of interventions “with the intent of producing new theories, artefacts, and practices that account for and potentially impact learning and teaching in naturalistic settings” (Barab & Squire, 2016, p. 2). The approach aims to create practical solutions to educational problems, generate transferable design principles, and contribute to existing theoretical knowledge (Plomp & Nieveen, 2013; van den Akker et al., 2006).

The educational design research process typically involves iterative cycles of analysis, design, implementation, and evaluation (Aşık & Yılmaz, 2017; Gravemeijer & Cobb, 2006). It begins with user needs and integrates user perspectives throughout (Schön, 1983) to generate tested design principles and activities. Design research also grounds PD development in existing scholarship and connects theory with practice in context (Barab & Squire, 2016; Plomp & Nieveen, 2013). It iteratively (a) tests and refines design principles and activities based on user insights and existing frameworks and (b) identifies effective elements of the PD and areas needing improvement. This process yields knowledge in the form of transferable design principles to inform the design of PD programs for learners across diverse contexts.

We engaged principals and PD experts early in the process, supporting the coconstruction of our design principles and learning activities that are responsive to school contexts, leadership needs, and mathematics learning goals. In what follows, we share the design principles and learning activities created for our AIM-SLP.

The AIM-SLP Design Principles

Based on our review of the literature, interactions with school principals, and the goals of the AIM-PD for teachers, we identified four key design principles for developing effective PD programs for principals focused on promoting high-quality mathematics discourse in the classroom:

- Draw on what is known regarding principals’ essential knowledge and practices to set PD goals that promote high-quality mathematics discourse.
- Align principal and teacher learning goals to select content that can support a schoolwide vision for high-quality mathematics discourse.
- Include principal educators in the design process to create feasible and relevant programs.
- Include practical, job-embedded tools that make the PD useful and easily applied to principals’ practice.

AIM-SLP Activities Design

AIM-SLP is a fully asynchronous, job-embedded PD initiative designed to empower school leaders to support

and enhance high-quality mathematics discourse in K–2 classrooms. In designing the AIM-SLP, the Project AIM team capitalized on our review of the literature, particularly the four design principles that emerged from this review.

Design Principle 1: Draw on What is Known

The AIM-SLP design emphasizes instructional leadership by equipping school principals with knowledge and practices necessary to foster a school culture grounded in a shared vision and goals for high-quality mathematics discourse. The program prioritizes fostering high-quality mathematics discourse and recognizes it as a key driver of student engagement, conceptual understanding, and productive learning opportunities. To operationalize this vision, AIM-SLP is structured around a set of clearly defined goals that guide school leaders in implementing effective leadership practices:

- Learn what constitutes high-quality mathematics discourse for all students in elementary classrooms.
- Understand how classroom discourse supports opportunities for all students to learn mathematics concepts, develop procedural fluency, and build productive dispositions.
- Prepare for and practice supporting teachers and coaches in their work to improve mathematics instruction that engages each and every learner in high-quality mathematics discourse.
- Develop practices to strengthen school culture for improving mathematics instruction.

Design Principle 2: Align Principal and Teacher Learning Goals

AIM-SLP content is designed to align school leader learning with foundational goals and ideas from AIM-PD for teachers, focusing specifically on high-quality mathematics discourse and instructional techniques to promote it (Sztajn et al., 2021). Modules in AIM-SLP deepen leaders’ understanding of how effective classroom discourse creates opportunities for students to develop conceptual understanding, procedural fluency, and productive dispositions. These modules also explore discourse techniques for supporting all learners, equipping principals to recognize and promote effective instructional practices. Through this alignment, AIM-SLP ensures school leaders are prepared to support teachers and coaches in implementing instructional strategies that engage all students in high-quality mathematics discourse.

AIM-SLP is comprised of eight modules progressing from foundational knowledge building to practical application. Modules 1–5 lay the foundation, building essential knowledge necessary for leading instructional improvement. These modules introduce Project AIM’s goals and structure, establish a vision for HQMI and discourse, and explore resources that support effective instructional practices for teachers. Modules 6–8 shift the focus to application, providing leaders with tools and strategies to foster teachers’ professional growth and establish a collaborative school culture centered on mathematics improvement. The overall structure of AIM-SLP is shown in Figure 1.

Figure 1
AIM-SLP Structure

Self-Guided Learning Modules					Academic Year Practice Modules		
1	2	3	4	5	6	7	8
School Leader Program Overview	What is Project AIM? What is My Role?	A Vision of High-Quality Math Discourse for All Students	Engaging Emergent Multilingual Learners in Math Discourse	Preparing for Practice	Supporting Teachers' Professional Learning	Supporting Teachers and Coaches to Improve Math Instruction	Enhancing School Culture to Promote High-Quality Math Instruction for All Students
Knowledge Building					Application to Practice		
Tools Resources for Supporting Teachers' Math Instruction							

Design Principle 3: Include Principal Educators

The AIM-SLP design recognizes the challenges (e.g., time, resources) faced by school leaders; its design features were carefully created in partnership with principal educators who understood these challenges and promoted realistic expectations for principal engagement. Thus, the self-guided nature of the program allows principals to progress through the content at their own pace and accommodates principals' demanding schedules. The eight modules of the program are designed to be completed in a total of approximately 8–12 hours during the school year, with the more engaging and time-intensive modules (i.e., Modules 1–5) intended for summer months when schedules are more open.

AIM-SLP also intentionally incorporates a variety of relevant and flexible instructional activities tied closely to principals' everyday leadership roles and responsibilities (e.g., classroom observations) and provides access to foundational content and skills needed to support teachers' implementation of HQMI and discourse practices. Sample activities include observing portions of the AIM-PD, studying videos of classroom instruction, responding to reflection prompts, and engaging with pertinent readings. These activities support principals in (a) identifying their needs, (b) assessing their own growth in knowledge, (c) setting actionable leadership goals to support HQMI and discourse, and (d) adapting what they learn to their unique school contexts. For instance,

in interactive video activities, principals enhance their professional learning on instructional leadership. The video platform pauses the videos at strategic moments of classroom instruction or teacher PD to present embedded reflection questions that prompt principals to consider how to respond at key moments or to envision how important ideas apply in their own leadership contexts.

Design Principle 4: Include Practical, Job-Embedded Tools

AIM-SLP supports principals in enacting practices directly in their school contexts. Resources such as the discourse analysis record sheet, classroom walkthrough checklist for use during observations of instruction, and conversation guides for discussions with teachers (see Figure 2 for an example of a conversation guide on reflecting on a mathematics lesson) were designed to integrate easily into leaders' daily routines. The program minimizes added workload while maximizing instructional impact by embedding these tools in existing leadership practices. These resources support principals in conducting focused observations, facilitating reflective discussions, and providing constructive feedback to drive instructional improvement. A resource library provides ongoing access to these tools so principals can use them to sustain efforts for promoting high-quality mathematics discourse after the program's completion.

Figure 2

Conversation Guide: Reflecting on a Mathematics Lesson

Project AIM PD: School Leader Program

Project AIM School Leader Resource
Conversation Guide: Questions for Reflecting on a Mathematics Lesson

After a teacher has taught a lesson aimed at promoting responsive discourse (whether you observed the lesson or not), you can use a subset of these questions as a guide to support the teacher's reflection. The lead-in questions will help you get a sense for which parts may be appropriate for the particular lesson.

Ask parts A and B to discuss the goals and task(s) of the lesson.

A. Setting Mathematics Content and Discourse Goals

Lead-in: Can you say a little bit about mathematics and discourse goals for the lesson?

B. Selecting a Task for Promoting Responsive Discourse

Lead-in: Can you describe your planning process for selecting the lesson task(s)?

1. How well did the lesson task(s) engage students with the mathematics and discourse goals?

Choose ONE of part C, D, or E to focus on a particular phase of the lesson.

C. Launching the Task <i>Lead-in: What considerations did you keep in mind when launching this task?</i> 1. In what ways did your launch prepare all students, including emergent multilingual learners, to engage with the task? What wasn't as effective or didn't prepare all students to engage? [You might follow up about: the context of the task, scaffolding questions, word walls/banks, encouraging various modes of communication]	D. Exploring the Task & Rehearsing to Share <i>Lead-in: How did you organize students for learning in this lesson so that all had opportunities to participate in responsive discourse?</i> 1. How well did the organization actively engage all students, including emergent multilingual learners, with Exploring the mathematics of the task? With engaging in responsive discourse through their [questioning, explaining, listening, and using various modes of communication]?	E. Discussing and Making Sense of the Mathematics of the Lesson <i>Lead-in: How did the lesson enable all students to further their mathematical thinking?</i> 1. How well did the Discuss support responsive discourse for all students, including emergent multilingual learners, in the ways they [questioned, explained, listened, and used various modes of communication]? 2. How well did the lesson make use of multiple solutions, including incorrect ideas, to deepen students' mathematics understanding?
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Ask part F to look ahead from this lesson to improving future instruction.

F. Looking Ahead to Future Instruction

1. What did you learn from this lesson that you want to remember for future instruction?
2. What would you like to improve for future instruction? What can the school do to support you in making those improvements?

STUDIES

To examine the relevance of AIM-SLP design principles and activities, we conducted three small, focused studies. Each study explored a distinct aspect of the program design and its implementation. The first study examined the alignment between our program design principles and what school leaders identified as essential and needed for support of high-quality mathematics discourse. The second study explored how the AIM-SLP activities reflect the recommendations expressed in the EA framework for leadership in mathematics education (NCSM, 2020). The last study examined how principals participating in the AIM-SLP perceived its impact on their instructional leadership practices. Together, these studies provided insight into how the program supports school leaders, informed revisions and refinements of the program, and offered design principles that can be shared with the broader field to guide similar professional learning efforts.

Study 1: AIM-SLP Design Principles' Alignment With Principals' Needs

Study 1 aimed to answer the first research question: In what ways do the PD principles derived from the literature align with what school leaders consider the knowledge, skills, and resources needed to implement high-quality mathematics discourse? To inform the design of AIM-SLP, the research team conducted a focus group with school leaders who were familiar with Project AIM and had teachers participate in the PD ($n = 5$). The focus group interview was aimed at understanding (a) what principals need to know about HQMI and discourse, (b) leadership practices for supporting teachers and cultivating a school culture that promotes high-quality mathematics discourse, and (c) types of tools and resources that are most helpful to principals in promoting high-quality mathematics discourse. We also inquired about the benefits and challenges of the proposed AIM-SLP PD, including its anticipated length and feasibility of a self-paced model.

The focus group data were analyzed to examine how school leaders expressed needs and insights on supporting high-quality mathematics discourse aligned with the foundational principles guiding the AIM-SLP development. Our findings are organized around each of the design principles.

Design Principle 1: Draw on What is Known

Analysis of the focus group interview data showed principals need a clear and shared knowledge and understanding of what constitutes effective mathematics instruction and productive mathematics discourse. This finding affirmed the first AIM-PD design principle of SLP. As Principal Becky explained:

I think just an underlying understanding, first of all, what effective mathematics instruction is and isn't. So, there needs to be sort of a basis for that . . . the supporting pieces for mathematics, this mathematical discourse. What does that look like? Why is it effective? Why is it important?

In addition to the knowledge of high-quality mathematics discourse, principals emphasized the need for specific

leadership skills, particularly the ability to engage teachers in reflective conversations that promote professional growth. As Principal Jack shared, "Principals need to be comfortable asking questions and learning from their teachers." Similarly, Principal Jennifer expressed, "The leadership skill is asking the right questions to get your teachers to reflect," underscoring the role of strategic inquiry in supporting teacher learning. These insights aligned with research recommendations (e.g., Grissom et al., 2021; Kazemi & Resnick, 2020; Kazemi et al., 2022) and reinforced the importance of grounding PD design in leadership knowledge, skills, and practices that directly support principals' instructional leadership for teacher reflection and growth.

Design Principle 2: Align Principal and Teacher Learning Goals

Leaders also emphasized the principals' role in developing collaborative learning spaces that ensure coherence in mathematics instruction across grade levels. They noted principals need to be aware of the focus of teacher learning to align leadership practices to improve discourse and HQMI. This insight validated the principle of aligning principal and teacher learning goals. For instance, Principal Jeremy highlighted the need for principal PD aligning with teacher PD for better supporting teachers, saying:

I think that with our first round, I felt like our first challenge was that we didn't know how to support our teachers through this professional development, because we didn't have [PD] . . . I was envisioning this [SLP] being paired alongside our teachers in Project AIM . . . simultaneously as they were attending [the AIM-PD].

Similarly, Principal Becky emphasized the need for aligning principals' learning with the teachers' learning. She said:

Learning along with your teachers, learning something that you really haven't seen, that you weren't trained in, that you probably didn't really practice in your own classrooms back in the day. So, you know, learning it to see what that looks like, and then being willing to engage with it with your teachers.

Design Principle 3: Include Principal Educators

Principals offered valuable input about the knowledge, skills, and practical tools they considered most useful and relevant in the SLP, and their input aligned directly with recommendations provided by the principal educator who was part of our team. The principals emphasized that program modules should be connected to classroom observations, manageable in length (i.e., no more than 1–1.5 hours per module, totaling under 10 hours), and include flexibility by offering foundational and optional content to meet the different stages of leadership development. Principals also suggested structuring the program to (a) be more engaging in the summer, when they have more time; (b) include a midyear virtual check-in with peers; and (c) culminate in a final spring session to reflect on progress, identify needs, and share experiences. These insights from the principals aligned with the recommendations, content,

and structure our partnering principal educator advised for the AIM-SLP.

Design Principle 4: Include Practical, Job-Embedded Tools

Principals indicated they needed to understand and have the necessary resources to conduct intentional observations in classrooms and to provide meaningful feedback to teachers. These recommendations supported our fourth principle of including practical, job-embedded tools. For instance, Principal Jennifer shared:

What should I expect to see in that mathematics classroom? Almost like, what, if our district's working on our walkthrough instrument for mathematics, what would be some things that we would put on there to make sure that we're seeing?

Principal Jennifer's reflection validated our design principle of including job-embedded, practical tools that help principals identify characteristics of HQMI and discourse as part of their classroom observation process. Similarly, Principal Jack emphasized the usefulness of resources aligned with teacher PD, sharing, "the *Activating Math Talk* book [Sztajn et al., 2021] is really helpful. We're doing some work with that as a follow-up to implementing Project AIM and [Cognitively Guided Instruction], and that's been a great resource for us to use." Principal Jeremy also suggested the program should provide some guiding questions to engage in reflective conversations with their teachers and said, "These are some ways that you could support them. Here's the critical pieces that you need to know. Here's some great questions that you could ask."

Study 2: NCSM EAs for Guiding Principles Alignment for AIM-SLP

Study 2 aimed to answer the second research question: In what ways do the designed AIM-SLP activities align with the EA framework for leadership in mathematics education outlined by the NCSM? Conducted by Project AIM's external evaluator, this study included a content analysis by aligning the AIM-SLP program's content with EAs in NCSM's (2020) framework for leadership in mathematics education to identify sources of evidence for potential alignment.

The framework records emerging themes in mathematics leadership through leadership guiding principles and indicators within each principle (see Table 1). These indicators embody "big ideas" and stages mathematics education leaders are likely to experience as they grow and evolve into their positions (NCSM, 2020), from a focus on or commitment to the leadership of the self to the leadership of others and the leadership of the extended community. The framework can be used to identify the knowledge and skills that might be targeted in PD programs for education leaders.

In the AIM-SLP alignment study, the project evaluator translated Table 1 into a spreadsheet that listed the EA for each guiding principle in rows and headed the columns with the AIM-SLP activities from the learning management system. Alignment of the contents of the activities to each EA was then scored in the resulting cells. One Project

AIM researcher also independently scored the contents of the activities in five selected modules to each EA. The discrepancies were resolved through consensus discussion. Table 2 shows a snapshot of the analysis spreadsheet.

We reviewed each AIM-SLP activity to consider its potential to provide leaders an opportunity to engage with key concepts of the EAs. The score in each cell in Table 2 represents the extent to which all instructional elements comprising the activity provided an opportunity to learn about what the EA meant in the context of Project AIM. Score definitions are:

- 0: the key concepts of the EA were not addressed in the activity
- 1: the key concepts of the EA were mentioned in the activity
- 2: the key concepts and/or skills of the EA were explained and connected to leadership activities
- 3: the key concepts were accompanied by an activity in which the principal had the opportunity to practice the knowledge and/or skills of the EA.

The matrices of the analysis indicated almost all EAs were at least mentioned in the text, readings, or video clips comprising the AIM-SLP content. About one third of the EA subsections were addressed through an explicit, required activity. Such activities provided participants with opportunities to learn about the appropriate knowledge and skills through reflection or action (e.g., observing some of the AIM PD activities with participating teachers). For example, Activity C1 in Table 2 required participating principals to review the professional norms for examining teacher practice, view a video of classroom instruction, and use the project's Math Discourse Matrix (Sztajn et al., 2021) to analyze the nature of the discourse they observed. The activity asked them to consider what they noticed about the nature of student mathematics discourse, the teacher's instruction, and how the discourse shaped students' opportunities to learn the mathematics content.

EAs for NCSM Guiding Principle 1

The analysis showed AIM-SLP consistently provided strong opportunities for principals to learn and engage with EAs related to Guiding Principle 1. EA1.1, EA1.2, and EA1.3 received some of the highest and most consistent scores across multiple activities, particularly in AIM-SLP Module 3: A Vision of High-Quality Math Discourse for All Students. This finding indicates the program promotes a shared vision of HQMI and actively supports school leaders in developing the knowledge and practices necessary to foster this vision in their schools. Activities associated with these EAs frequently went beyond mentioning or explaining concepts by offering opportunities for leaders to practice and reflect, which was noted as a strength of the program.

EAs for NCSM Guiding Principle 2

EAs connected to Guiding Principle 2 showed varying levels of emphasis across the program. Although these components received lower scores in earlier modules, they demonstrated stronger alignment in Module 5: Preparing for Practice and Module 6: Supporting Teachers' Professional

Table 1*Structure of the NCSM Framework for Leadership in Mathematics Education*

Guiding principles	Big ideas	EA
1. Advocate and expect high-quality, equitable mathematics teaching and learning for every student	Curriculum leadership Instructional leadership	1.1: Ensure every teacher possesses a shared understanding and vision of HQMI and the actions required to meet the vision. 1.2: Ensure every teacher possesses the skills and knowledge necessary to design and implement meaningful learning experiences that lead to student understanding of mathematics. 1.3: Ensure all stakeholders have a clear understanding of HQMI and how to support it.
2. Design and implement structures that support high-quality mathematics teaching and learning by every teacher	Structures that support equity Structures supporting teachers' work with students Structures supporting teacher professional learning	2.1: Ensure mathematics learning for all students through organizational structures, time and resource allocation, and systemic supports that are aligned, intentional, and equitable. 2.2: Ensure systems of continual collaborative, job-embedded professional learning to build teacher and leaders capacity and increase efficacy. 2.3: Ensure sustainability through engaging all stakeholders in systemic, long-range strategic planning for all teaching and learning improvement initiatives.
3. Empower and nurture a culture of productive professionalism	Beliefs and mindsets Building relationships Culturally sustaining practices	3.1: Ensure assumptions, beliefs, expectations, and habits are examined to shape the school or department culture around teaching and learning of mathematics. 3.2: Ensure a culture of reflection, refinement, and action focused on continuous improvement in mathematical learning. 3.3: Ensure students, teachers, families, and community partnerships are built on meaningful engagement.
4. Monitor and act on evidence of student learning	Classroom level Teacher teams District level	4.1: Ensure the design and use of high-quality, aligned assessments and equitable assessment processes that guide meaningful reflection and action. 4.2: Ensure that the evidence of learning collected from every assessment is used to inform the design of curriculum, instruction, and the assessments themselves. 4.3: Ensure every student is provided access to grade-level content and intensification based on evidence of student learning.

Table 2*Snapshot of a Portion of the Analysis Spreadsheet*

EAs	Project AIM-SLP Module 3 components				
	Discourse	HQ		Analyze discourse	
	A1: Watch	B1: Read	B2: Reflect	C1: Examine	C2: Review
1.1: Ensure every teacher possesses a shared understanding and vision of HQMI and the actions required to meet the vision.	1	1	2	3	1
1.2: Ensure every teacher possesses the skills and knowledge necessary to design and implement meaningful learning experiences that lead to student understanding of mathematics.	1	1	2	3	1
1.3: Ensure all stakeholders have a clear understanding of HQMI and how to support it.	1	1	2	3	1

Note. From Module 3: A Vision of High-Quality Math Discourse for All Students.

Learning, where they consistently achieved ratings of 2–3 across multiple activities. This pattern suggests an intentional program design that first establishes instructional leadership foundations before transitioning to the organizational structures needed to sustain improvement. The higher ratings in Modules 5 and 6 indicate that AIM-SLP addresses these critical systemic elements, particularly as participants move toward practical implementation in their school contexts.

EAs for NCSM Guiding Principles 3 and 4

EAs associated with Guiding Principles 3 and 4, including cultivating beliefs, culturally sustaining practices (i.e., EA3.1 and EA3.2) and leveraging assessment data to monitor students' learning (i.e., EA4.1 and EA4.2), were less frequently and deeply addressed across program activities. Although some elements related to reflection (e.g., EA3.1 through ways for collaborating with mathematics coaches and teachers to build reflective culture) and culture building (e.g., EA3.3 through emphasizing importance of communicating with caregivers and sharing caregiver newsletter resources) were present in the activities, there were limited opportunities for principals to actively practice strategies and use the tools in these areas. One particular set of EAs was somewhat problematic to judge in terms of alignment with the AIM-SLP, namely the following EAs under GP4: Monitor and act on evidence of student learning:

- EA4.1: Ensure the design and use of high-quality, aligned assessments and equitable assessment processes that guide meaningful reflection and action.
- EA4.2: Ensure the evidence of learning collected from every assessment is used to inform the design of curriculum, instruction, and the assessments themselves.

The text in the NCSM leadership documents focuses primarily on formal and summative assessments rather than the very formative assessments emphasized in Project AIM for initiating and supporting classroom discourse in K–2 mathematics lessons. Specifically, formative assessment is a core element of Project AIM because it informs the teachers' moment-by-moment instructional decisions about classroom discourse. Accordingly, the Project AIM evaluator concluded, the lack of evidence for EA4.1 and EA4.2 should not be construed as a critique or weakness of the AIM-SLP. Rather, it is appropriate that the formative assessment in the AIM-SLP addresses providing every student with access and practices aligned to in-the-moment instructional decision making and proximal steps of classroom instruction. In fact, the third EA under GP4, EA4.3: Ensure every student is provided access to grade-level content and intensification based on evidence of student learning, was mirrored throughout the AIM-SLP activities and was therefore judged to align well with the instructional elements provided to participating principals.

Study 3: Principals' Perspectives

Study 3 aimed to answer the third research question: What are the perspectives of participating principals regarding the contributions of the AIM-SLP to their support for high-quality mathematics discourse? Through the analysis

of principals' perspectives, we explored how AIM-SLP experiences contribute to their practices to lead, support, and sustain high-quality mathematics discourse in their school settings.

Two principals, Ayanna and Sarah, engaged in the AIM-SLP beginning in Summer 2023 and continuing throughout the 2023–2024 school year. Ayanna and Sarah worked for the same school district in the southeastern region of the United States. The district was in a small city setting with 11 elementary schools and served nearly 12,000 students, of which 54% were from racially/ethnically underrepresented populations and 48% were economically disadvantaged. Ayanna had been the principal at the same school for 7 years. Ayanna's school was a PK–5 public elementary school serving nearly 350 students, of which 44% were from racially/ethnically underrepresented populations, and 31% were economically disadvantaged. Sarah had been her school's principal for 9 years. It was also a PK–5 public elementary school serving a slightly larger population of nearly 400 students, of which 54% were from racially/ethnically underrepresented populations and 48% were economically disadvantaged. Both Ayanna and Sarah had been assistant principals of their school prior to becoming the head principals.

To capture principals' perspectives and experiences participating in AIM-SLP, we collected Ayanna's and Sarah's written responses to key SLP activities documented in the learning management system and interviewed each principal three times spanning their participation (initial, follow-up, and final). The process was as follows:

- The initial interview focused on principals' early impressions, key takeaways from the first half of the program, and feedback on specific activities and tools in Modules 1–5. This interview lasted 30–55 minutes.
- The follow-up interview, conducted at the end of the 1st semester of the 2023–2024 academic year, explored principals' learning about leadership practices from Modules 6–8. It elicited their successes and challenges in applying ideas and practices from AIM-SLP modules in their schools and lasted 30–45 minutes.
- The final interview, conducted at the end of the 2nd semester of the 2023–2024 academic year, provided principals with an opportunity to reflect on the AIM-SLP's impact on their leadership practices and knowledge. It lasted 20–45 minutes.

We employed a thematic analysis approach drawing from multiple data sources (i.e., transcripts of interviews with the principals and their responses to key activities in AIM-SLP). We used a combined technique of inductive and deductive thematic analysis in a collaborative coding process (Fereday & Muir-Cochrane, 2006; Richards & Hemphill, 2018). The deductive data analysis was guided by selected practices from the unified model of effective leadership practices (Hitt & Tucker, 2016) to explore how AIM-SLP supported these practices. Meanwhile, two researchers conducted an inductive analysis in areas where the framework did not fully capture principals' perspectives. Any discrepancies in coding were resolved through discussion until consensus

was reached. Table 3 presents the final coding scheme for the study.

Table 3
Coding scheme: Effective Leadership Practices

Practice domain	Practices
Establishing and conveying the vision*	Communicating broadly the state of the vision Setting expectations and goals*
Building professional capacity	Providing opportunities to learn Engendering responsibility for learning
Creating a supportive organization for learning	Strengthening and optimizing school culture Sharing and distributing leadership
Teaching and learning environment	Providing instructional support: • Having targeted and reflective conversation** • Intentional observations and walkthroughs** Developing a shared language***

Note. * Slightly revised language from original practice in the framework. ** Subpractice that is coded as a result of inductive analysis. *** Practice that is coded as a result of inductive analysis.

Our analysis showed AIM-SLP supported participating principals in four main leadership practice domains: (a) establishing and conveying the vision of HQMI, (b) building professional capacity, (c) creating a supportive organization for learning, and (d) enhancing the teaching and learning environment.

Establishing and Conveying the Vision of HQMI

Principals described how their participation in the AIM-SLP influenced their understanding about establishing and conveying the vision of high-quality mathematics discourse across grade levels. They shared their vision of sustaining AIM ideas (e.g., mathematics talk techniques), particularly as students transitioned from K–2 to Grades 3–5. The principals expressed the need for deliberate planning to ensure discourse practices developed in early grades would not be lost in subsequent years. Both principals highlighted the value of Project AIM's *Activating Math Talk* book (Sztajn et al., 2021) as a tangible resource for supporting consistent practices across grade levels.

Principals' takeaways arising from their work in the AIM-SLP also emphasized the importance of setting clear expectations and goals for high-quality mathematics discourse. They highlighted fostering meaningful student engagement through student-led discussions, selecting discourse promoting tasks, and supporting responsive discourse (Sztajn et al., 2021) in mathematics classrooms as critical expectations. Principals recognized the need

to ensure such practices become embedded daily in mathematics teaching. For instance, Ayanna noted:

I always say to my staff, whoever's doing the talking is doing the learning. If you walk into my math classes, you'll see kids talking. . . . How do you have kids adding on and really leading the discussion? . . . These strategies [referring to AIM discourse techniques] are part of the expectation. What we've learned in AIM should show up in lesson plans.

Building Professional Capacity

Analysis of principals' interview and program reflection data revealed two types of practices for providing opportunities to learn. One practice involved principals focusing on their own learning as instructional leaders. They shared AIM-SLP's expectation that principals participate in parts of the AIM teacher PD sessions alongside their teachers to help them learn ideas (e.g., AIM's mathematics discourse techniques and discourse types) that support instructional leadership. For instance, Sarah articulated, "Whenever we're learning something new, that 1st year, I go through the learning process with my staff. I do it all with them. The 2nd year, I start giving feedback and implementing because I feel more equipped."

The principals also explained their plans for expanding and sustaining opportunities for professional learning for their teachers related to AIM ideas in their schools. For instance, Sarah shared:

The biggest takeaway has been having time and space for people to reflect on their practice, not just by yourself, but as a group, as a team. To do a thing well—to get information about it, reflect on it, and then engage in dialogue—helps us move forward.

Principals also noted that AIM-SLP supported them in cultivating a growth mindset among teachers while engaging with AIM-related learning opportunities. They especially acknowledged their role in engendering responsibility for learning by encouraging and creating spaces for teachers to implement new mathematics talk techniques in their classrooms. From her experience, Ayanna recounted:

Anytime there's something new, I think there's this uneasiness like, "Am I doing it right?" And I always tell them, "You're not gonna harm the kids trying. Harming kids is not trying something or not teaching them." When they first do a new technique, they feel uneasy. . . . We talk about being uncomfortable . . . and it's okay. 'Cause that's when the learning takes place.

Creating a Supportive Organization for Learning

Both principals, Ayanna and Sarah, said after participating in the AIM-SLP, they focused on strengthening and optimizing school culture to create a coherent approach to high-quality mathematics discourse across grade levels. They indicated their plans for several professional learning opportunities to ensure AIM-related learning reached teachers across K–5 grade levels (e.g., coaching cycles, book clubs focusing

on *Activating Math Talk*, peer observations). They aspired to reinforce and integrate AIM concepts schoolwide, ultimately establishing a consistent and effective approach to mathematics discourse in their schools' classroom culture.

Sarah appreciated how AIM-SLP influenced her leadership approach, shifting from a general focus on school culture to a more intentional emphasis on continuous improvement and targeted growth areas. She elaborated:

What are we going to do as a school? Through coaching cycles, how do we apply this? What does it mean for third, fourth, and fifth grades as we onboard them with the information? . . . It's about narrowing focus on a culture of improvement rather than just overall school culture.

Sarah also identified strategies for strengthening school culture by fostering engagement with parents and caregivers (e.g., incorporating AIM-SLP resources for family mathematics night activities, using caregiver newsletters to promote ongoing communication and productive mathematics talk at home).

The principals also acknowledged that creating school culture requires a collaborative effort. They found the AIM-SLP supported them in using strategies for distributing leadership responsibilities among staff (e.g., coaches, teachers). Ayanna described her plans to use specific tools and resources from the AIM-SLP to share leadership responsibilities with her school's instructional coach. Sarah specified her plans, saying:

It's myself, coaches, and some key person in the grade levels. One of the other things that's going to be quite instrumental is having a teacher on each of the grade levels who has experienced the AIM work and is excited to share the work. While the coach is going to be coaching the grade level, having the teacher also take leadership roles.

Enhancing the Teaching and Learning Environment

The principals shared the AIM-SLP supported them in three specific ways to provide instructional support for their teachers to enhance the teaching and learning environment. First, they started to engage in more targeted conversations with teachers during lesson planning, lesson observation reflections, and post-PD reflections. They used AIM's conversation guides, the Questions to Ask Yourself resource, and the Math Discourse Matrix (Sztajn et al., 2021) to ask specific questions and identify key ideas to push teachers' thinking and improve instruction. For instance, Sarah noted, "[The AIM-SLP] helps me to push thinking and to be able to ask a more specific question. So, being able to have very targeted questions or targeted look fors."

The principals also shared their intention to use AIM-SLP's tools and resources to engage in more targeted conversations—not only with teachers, but also with instructional coaches. For example, Sarah began incorporating conversation guides into her meetings with coaches and said, "Now, I feel more targeted in my conversations because, earlier, some of my conversations have been around what's been happening, but this just gave me more knowledge."

Second, Sarah and Ayanna emphasized that AIM-SLP supports them in conducting intentional classroom observations aimed at identifying specific areas for teacher growth. They recognized AIM-SLP resources (e.g., conversation guides, Questions to Ask Yourself), along with the discourse techniques, were helpful for the observations. Both principals also highlighted that AIM-SLP supports them in conducting intentional walkthroughs that move beyond broad reflections like, "What went well." Sarah and Ayanna planned to use the AIM-SLP's walkthrough checklist tool to gather key information about instructional practices and student discourse. They intended to use this information to provide specific instructional support.

Both principals acknowledged that developing a shared language is essential for providing intentional instructional support for teachers and fostering a collaborative school culture. They emphasized AIM-SLP's encouragement for principals to actively participate in AIM-PD with teachers helped them establish this shared language. Ayanna reflected:

So, I do think having some of that common language, so, when we're in a meeting in PLCs [professional learning communities], if they're not talking about it, I can remind them. When I'm doing observations, I can just say, "Hey, this could have been a time where you used this strategy."

Additionally, Sarah noted the AIM-SLP gave her the knowledge and confidence to engage in productive instructional conversations with teachers. She was better able to connect with teachers' experiences by developing shared language, which strengthened her instructional leadership, built trust, and increased teacher buy-in.

DISCUSSION AND IMPLICATIONS

This study addressed the need for mathematics-focused principal PD by designing the asynchronous, job-embedded AIM-SLP aligned to teacher learning (Kazemi & Resnick, 2020) and aimed at supporting school leaders in fostering high-quality mathematics discourse as a key component of HQMI (Munter, 2014). The AIM-SLP offers an intentional model for school leaders' professional learning grounded in four design principles aligned with the well-established framework for leadership in mathematics education (NCSM, 2020) and supported by early evidence of relevance and usability among school principals.

The findings from three focused studies suggest: (a) strong alignment between principals' expressed needs and the AIM-SLP design principles; (b) consistent alignment of AIM-SLP activities with NCSM EAs for GP1 and GP2, with lighter emphasis on EAs for GP3 and GP4; and (c) support throughout AIM-SLP for leadership practices in four domains. Collectively these results suggest AIM-SLP is a promising approach for supporting school leaders to cultivate high-quality mathematics discourse as a key component of HQMI.

The results from these studies also carry important implications for those designing PD for school leaders. One implication is the importance of creating intentional

opportunities for leaders and teachers to learn together. Alignment between AIM-PD and AIM-SLP creates a powerful, shared learning model. When principals engage with the same tools and language as their teachers, their instructional support becomes more relevant and connected to classroom practices. This shared learning not only strengthens the coherence of PD across teachers and school leaders, but also allows leaders and teachers to work toward a shared vision and goals of instructional improvement in mathematics. In addition, the development of shared language extends beyond strengthening instructional feedback or recognizing high-quality mathematics discourse in K–2 classrooms for school leaders. Shared language fosters a culture where teachers, coaches, and school leaders can all contribute to schoolwide improvement and encourages broader engagement in promoting discourse-rich mathematics instruction. It also supports a clear instructional vision with high expectations that everyone can understand and uphold.

The second implication is the value of the tools that are job-embedded and easy to enact. School leaders need resources that help them notice, reflect, and respond to HQMI in real time. Practical tools (e.g., conversation guides, walkthrough checklists) shared in the AIM-SLP served as boundary objects that enabled principals to ground their conversations, observations, and feedback in a common framework. These tools became catalysts for principals moving from knowing to doing.

A third implication is that school leaders' appreciation of high-quality mathematics discourse as a key aspect of HQMI, developed through participation in AIM-SLP, prompted

them to plan for intentional and coherent scale-up from K–2 classrooms to K–5. A recommendation for future design of school leader PD programs, including AIM-SLP, is to include extensions that support leaders in building a coherent instructional vision and discourse practices spanning all elementary grades in the school setting. This could be an important first step toward creating coherence from the classroom to the district office for instructional improvement (Cobb et al., 2020).

One final implication relates to the partial alignment between AIM-SLP and EAs in NCSM's (2020) framework for leadership in mathematics education, particularly around formal assessment. This finding highlights the broader point that not every PD program should be expected to address all aspects of a comprehensive leadership framework. Rather than treating this as a limitation, our findings suggest a more nuanced view: frameworks are guides, not checklists. Embracing this perspective invites future researchers and PD designers to consider more adaptive approaches to both program development and evaluation, where alignment is examined thoughtfully in ways that capture the unique goals and contexts of each PD model.

The three studies we conducted are exploratory and have limitations. In particular, only two principals engaged in the AIM-SLP, and the evidence from the third study relied heavily on self-report of the principals' intended, rather than enacted, practice. In future research, we will examine how leaders' participation in AIM-SLP translates into realized changes in their leadership practice and, ultimately, into classroom instruction and student learning.

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