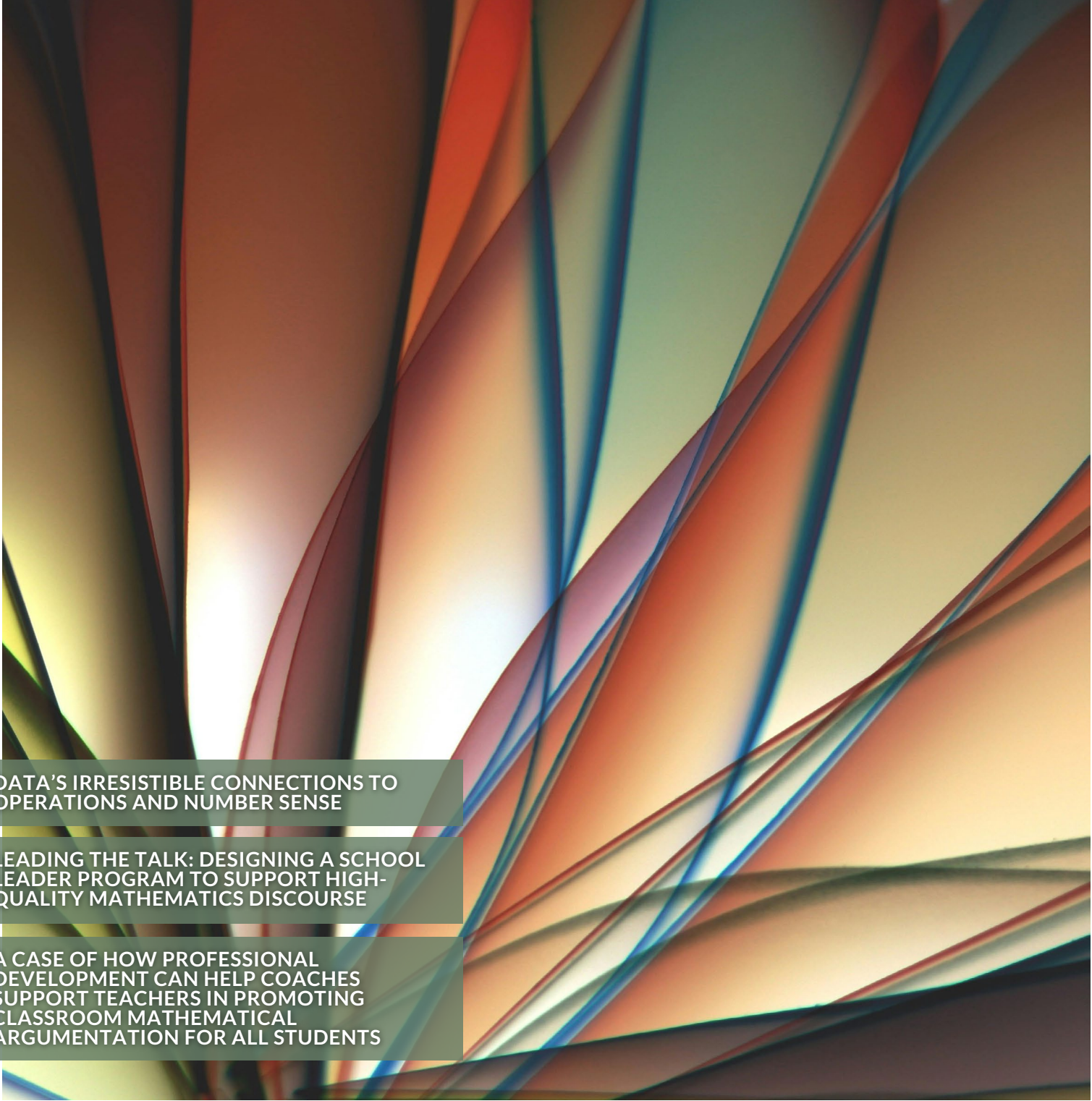


VOLUME 33 | ISSUE 2 | FALL 2026

NCSM JOURNAL

OF MATHEMATICS EDUCATION LEADERSHIP



DATA'S IRRESISTIBLE CONNECTIONS TO
OPERATIONS AND NUMBER SENSE

LEADING THE TALK: DESIGNING A SCHOOL
LEADER PROGRAM TO SUPPORT HIGH-
QUALITY MATHEMATICS DISCOURSE

A CASE OF HOW PROFESSIONAL
DEVELOPMENT CAN HELP COACHES
SUPPORT TEACHERS IN PROMOTING
CLASSROOM MATHEMATICAL
ARGUMENTATION FOR ALL STUDENTS

Table of Contents

04 ABOUT NCSM

05 COMMENTS FROM THE EDITORS

Heather Crawford-Ferre and Chadd McGlone

07 DATA'S IRRESISTIBLE CONNECTIONS TO OPERATIONS AND NUMBER SENSE

by Kat McKinney and Claudia Bertolone-Smith

This article discusses a workshop designed to enhance elementary teachers' understanding of how data, number sense, and operations are interconnected. Despite the important role of data in the Common Core State Standards, many teachers have reported feeling unprepared to teach data concepts. Accordingly, the workshop engaged teachers in hands-on activities (i.e., data blocks, datafied counting collections, graph-talk routines) designed to deepen their knowledge and confidence in teaching data. These activities helped teachers explore connections between data and other math concepts (e.g., comparison, counting, estimation). The article highlights the importance of viewing mathematics as an interconnected whole and offers strategies to integrate data-rich activities into classroom instruction, promoting deeper mathematical understanding and student engagement.

13 LEADING THE TALK: DESIGNING A SCHOOL LEADER PROGRAM TO SUPPORT HIGH-QUALITY MATHEMATICS DISCOURSE

by Zuhail Yilmaz, Paola Sztajn, Daniel J. Heck, Kristen A. Malzahn, Julia Schnegg, Lindsey H. Sachs, Edith Gummer, Francine P. Riddick

This article presents the design and evaluation of the Project All Included in Mathematics (Project AIM) School Leader Program (SLP), which supports principals in fostering high-quality mathematics discourse. We used design research to conduct three studies examining (a) SLP's design principles, (b) activity alignment with the National Council of Supervisors of Mathematics (NCSM) essential actions (EA) framework for leadership in mathematics education, and (c) principals' perspectives on SLP's contribution to their learning and leadership practices. Findings showed strong alignment between principals' needs and the design principles, consistent connections between SLP activities and EA, and key principals' perspectives, highlighting growth in instructional vision, capacity, and instructional support. Results also underscore the importance of aligning leader and teacher learning, embedding practical leadership tools, and approaching EAs as a guide when designing professional development (PD) for school leaders.

29 A CASE OF HOW PROFESSIONAL DEVELOPMENT CAN HELP COACHES SUPPORT TEACHERS IN PROMOTING CLASSROOM MATHEMATICAL ARGUMENTATION FOR ALL STUDENTS

by Anushree Bopardikar, Jennifer Knudsen, Christina B. Silva, Harriette S. Stevens

This article overviews how professional development (PD) can help coaches support teachers in fostering classroom mathematical argumentation (CMA) for all students. After summarizing conceptual underpinnings of a PD program on coaching, the article describes the program, presenting a case study of how structures and tools designed by the PD for lesson planning and reflection informed a coach's discourse with a teacher. Analyses of transcripts showed the extent to which a coach-teacher dyad discussed various topics related to mathematical concepts and practices and students' participation during early and late coaching cycles, informed by the purposes and protocols designed to visualize, plan, and debrief lesson enactment. The article highlights recommendations for PD design and research to support mathematics coaches.

50 INFORMATION FOR REVIEWERS

DATA'S IRRESISTIBLE CONNECTIONS TO OPERATIONS AND NUMBER SENSE

Kat McKinney
California State University
Claudia Bertolone-Smith
California State University

ABSTRACT

This article discusses a workshop designed to enhance elementary teachers' understanding of how data, number sense, and operations are interconnected. Despite the important role of data in the Common Core State Standards, many teachers have reported feeling unprepared to teach data concepts. Accordingly, the workshop engaged teachers in hands-on activities (i.e., data blocks, datafied counting collections, graph-talk routines) designed to deepen their knowledge and confidence in teaching data. These activities helped teachers explore connections between data and other math concepts (e.g., comparison, counting, estimation). The article highlights the importance of viewing mathematics as an interconnected whole and offers strategies to integrate data-rich activities into classroom instruction, promoting deeper mathematical understanding and student engagement.

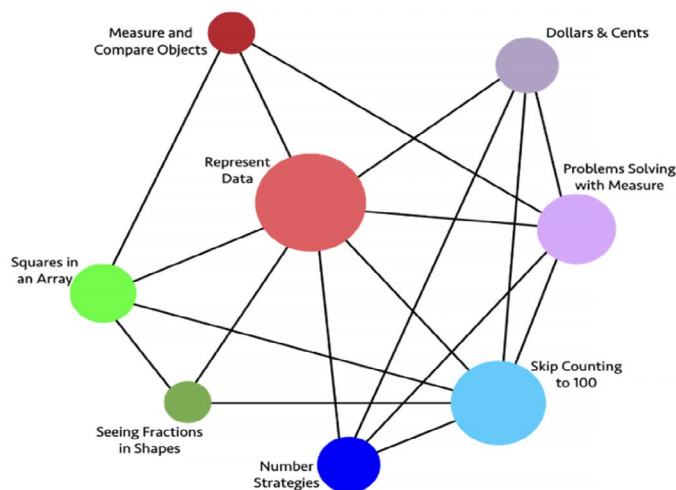
In Spring 2024, we—two scholar-practitioners—asked 44 elementary-grades teachers in northern California to respond anonymously to a survey about their teaching of data concepts. The following responses capture some of their sentiments: “I have not used data concepts outside of graphing,” “I feel that I have little understanding on what or how to teach data concepts at my grade level,” and “I don’t think I’ve thought about teaching data at all [this year].” The topic of data has served as a strand (paired with the topic of measurement) in all grade bands of the Common Core State Standards (National Governors Association & Council of Chief State School Officers, 2010) since the standards’ rollout in classrooms across the United States. Many elementary-grades educators, though, have viewed the topic of data as a tangential “if we have time” or add-on topic rather than an essential strand that holds rich opportunities for learning mathematics. We saw this sentiment expressed repeatedly in the survey results.

We also found that participating teachers expressed limited knowledge about elementary grades data topics and/or how to teach them. Indeed, this finding reflected a larger trend: Many practicing teachers nationwide do not have adequate content preparation in data and statistics (Franklin et al., 2014). Accordingly, the American Statistical Association’s *Statistical Education of Teachers* report argued for high-quality professional learning opportunities designed to help

in-service teachers advance their understanding of data and statistics (Franklin et al., 2014).

Fortunately, the notion that data topics are central to children’s mathematics education in all grades has gained traction in recent years. Notably, the California mathematics framework (California Department of Education, 2023) included at least one data-centric “big idea” in each grade. The framework highlights the link between data-focused big ideas and other big ideas via interconnected, grade-by-grade graphics, with the most connected ideas appearing larger and more centrally on the graphic. Figure 1 shows an example of an interconnected graphic from the California mathematics framework showing the big ideas of second-grade mathematics. At the center of the graphic, with the largest node, is the big idea to “represent data,” which directly connects to all seven of the second-grade big ideas (e.g., number strategies, measure and compare objects), indicating its conceptual link to each of those big ideas. Further emphasizing the importance of data topics, the framework lists “reasoning with data” as 1 of 4 cross-cutting “content connections” that span the mathematics curriculum from transitional kindergarten (TK) through Grade 12.

Figure 1
Connections Between the Big Ideas of Second-Grade Mathematics



Note. Adapted from *Mathematics Framework for California Public Schools* (p. 47), by California Department of Education, 2023 (<https://www.cde.ca.gov/ci/ma/cf/documents/mathframeworkch6.pdf>). Copyright 2023 by California Department of Education.

Beyond its connectedness to other topics, teaching mathematics through a data-focused lens can promote more equity and access for children. Indeed, data-centered activities often occur in contexts that include firsthand experiences for children and/or tap into their own funds of knowledge (González et al., 2001). Further, collecting and analyzing data can help students learn more about themselves and their worlds (Bargagliotti et al., 2020).

BACKGROUND

With these ideas in mind, we developed and gathered activities for a workshop we called Data ICONS (Data's Irresistible Connections to Operations and Number Sense). The 4-day workshop, which was cofacilitated twice by our research team, occurred in consecutive weeks of July 2024. Nineteen TK–5 teachers attended the first workshop, and 25 attended the second workshop. Teachers were from a range of counties, districts, and schools across northern California, with years of experience teaching ranging from 1 to more than 30.

Our primary goals for the workshop were to help the participating teachers (a) deepen understanding of the myriad connections between the topics of data, operations, and number sense through firsthand experiences as mathematics learners and (b) increase their confidence and knowledge of ways to incorporate data-centered activities in their own classrooms throughout the year. In this article, we share some of the key activities we used in Data ICONS for our teacher participants. We present these activities as part of three different collections representing three approaches for activity design that teachers can use to highlight connections between data, number sense, and operations. The collections, detailed in the following sections, are (a) data blocks, (b) datafied counting collections, and (c) graph-talk routines.

Data Blocks

Our first collection of activities was called data blocks. A data blocks activity entailed the use of common physical math manipulatives to represent individual numerical data points. For example, a data point with the value 85 could be represented by using place value blocks in the usual way (i.e., eight long sticks and five small cubes). In our workshop, we first used data blocks to represent each participant's data point generated in response to the question, "How old is the oldest person you've known?" (Bush et al., 2014). If the oldest person a participant had ever known was 105 years old, for example, then they used one flat hundred block and five small one cubes to represent the value of their data point. After everyone had a place value block representation of their data point, we proceeded with various data blocks activities designed to incorporate connections to number sense and operations.

One of our first data blocks activities connected data to number sense by way of comparing numerical values visually and conceptually. We asked teachers to put their place value blocks onto a small cookie sheet and meander around the room holding their trays. As participants encountered each other, they compared the values on their trays to the values

on other participants' trays without any participants saying the values aloud. By using visual comparisons of two values, participants either (a) bumped trays if the values happened to be equal or (b) briefly put the tray with the greater value above the tray with the lesser value. Figure 2 shows a pair of participants comparing their "oldest person" values in this way.

Figure 2

Two Participants Comparing the Values of Their Data Points



Note. In this example of two participants comparing the values of their data points (99 and 93), they briefly placed the tray of 99 above the tray of 93 to signify that its value is larger.

We then expanded on this value-comparing activity by having the whole group get in order from least to greatest value, again without saying their individual data value aloud to facilitate the visual and conceptual comparison of the blocks. When all participants were in a row from least to greatest data values, we considered various questions as a group to deepen our understanding of the data set. For example, we explored the following:

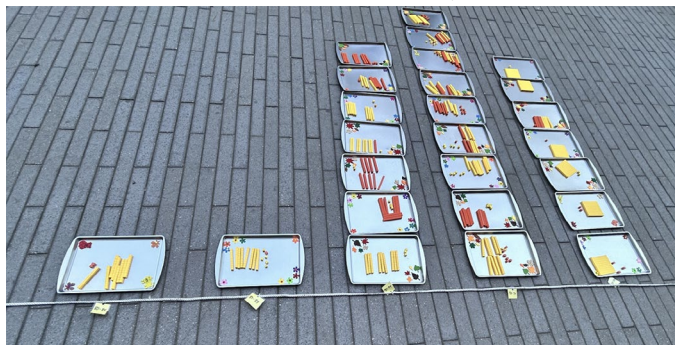
- Identifying our lowest data point and our greatest data point.
- Figuring out the difference between those values and how it related to the question, "How old is the oldest person you've known?"
- Stepping forward if a person and their neighbor (or multiple neighbors) had the exact same value. What did that mean?
- Counting simultaneously from both ends of the row to find the middle data point or points.

Next, we went outside, and participants placed their data block trays onto a number line made of rope. The rope was segmented into labeled ranges (e.g., 70–79, 80–89) to create a large bar chart representation of the data (see Figure 3). We considered new questions and ideas using this bar chart that connected our data to number sense and operations. We explored the following questions:

- Which age range has the largest number of data points, and how many data points are in that category? (Is this surprising? Why or why not?)
- Which age range has the least number of data points, and how many data points are in that category? (Is this surprising? Why or why not?)
- How could we use this chart to determine the total number of data points in our data set? (Let's do it! What strategy did you use to find the answer? What operation(s) did you use? How could we represent that strategy with an equation?)
- What would happen if our age ranges had different age spreads in them? For example, what if one category went from ages 70–74, and then the next category went from 75–79? Would this make sense to do, and why or why not?
- What would be the next age range up from our current greatest age range? How do you know? What would be the next age range down from our current lowest age range? How do you know?

Figure 3

Bar Chart Created From Data Blocks Trays



Note. This bar chart was created from a labeled rope and participants' data blocks trays.

Data blocks can be used to represent other types of numerical data, not just whole-number data points with multiple 10s. For example, fraction data points can be represented using fraction circles, pattern blocks, or fraction strips; whole-number data points with values less than 20 can be represented using snap cubes or physical ten frames with counters; and decimal data points can be represented with place value blocks by using a larger sized block as 1.

In our workshop, we used snap cubes to represent data points, all of which had whole-number values less than 10. To generate these data, we asked the statistical question, "How many peas are in a snap pea pod?" Each participant received a snap pea pod, opened it, and counted the number of peas in their pod. Then, each participant created a snap cube tower representing their number of peas. We then went through activities that mirrored those described previously for the oldest person data, with each teacher holding their

snap cube tower for the activities that called to move around the room (i.e., partner comparison; lining up in order from least to greatest; and analyzing data when in order least to greatest, including finding range, median, and mode).

Datafied Counting Collections

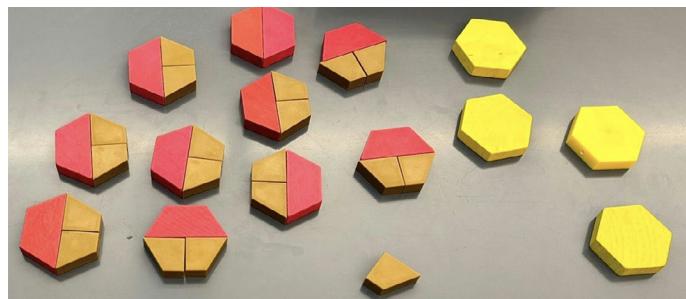
Counting collections (Franke et al., 2018) are a powerful and well-known math activity that can help students in any grade strengthen their number sense. The core of any counting collection activity is as follows: students, typically working in pairs, choose (or are given) a bag or box of items to be counted. The number of items in the collection might be small or large, depending on the student's grade level and proficiency with counting. With their partner, students empty the bags to display their collection and strategize on ways to count the total number of items. Tools to support counting are typically made available to students (e.g., empty cups or bowls, ten frames, hundred charts). After completing their count, students typically record their counting strategy on a recording sheet, making use of drawings, mathematical symbols, words, or a combination of these tools.

For our workshop, we developed activities that "datafied" the standard counting collection routine. In one activity, we gave pairs of participants bags consisting of different numbers of pattern block shapes. Their task was to count the number of each different shape in their bag. Each pair counted their collections and recorded their counting strategies on individual sheets. From there, each team created a graphical representation of their counts to represent their findings; for example, some participants created a bar chart to show the number of each shape in their collection. Participants used their graphs to explore several questions connected to the concepts of number sense and operations, including the following:

- Which shape did our collection have the most of? Which shape did we have the least of? What's the difference between the amounts of these two shapes? How can we find that difference on our graph?
- Did we have the same amount of any two of our shapes? How can we see that on the graph?
- Was there any shape that we had more than 10 of? How can we see that on the graph?
- If we combined the amounts of two of our shapes, would it be more than, less than, or equal to 20? How can we use the graph to show this?

In a subsequent datafied counting collections activity, we again gave each pair of teachers a bag of pattern blocks (with varying amounts of blocks in each bag). Then, in the style of a standard counting collection activity, we asked each pair to find the total "value" of their bag if the area of a yellow hexagon was worth 1. Figure 4 presents an example of one team's counting strategy.

Figure 4
Example Team Strategy for Finding Total Value



Note. This photo depicts one team's strategy for finding the total value of their collection (i.e., $14 \frac{1}{4}$) if the area of a yellow hexagon is worth 1.

After each participant recorded their counting strategy on their own paper, each team wrote the total value of their bag on the whiteboard at the front of the class, which then formed a data set. Together, using our class data set, we coconstructed a line plot with fractional values and proceeded to have a number talk (Parrish, 2010) about the difference between the bag with the largest value and the bag with the smallest value.

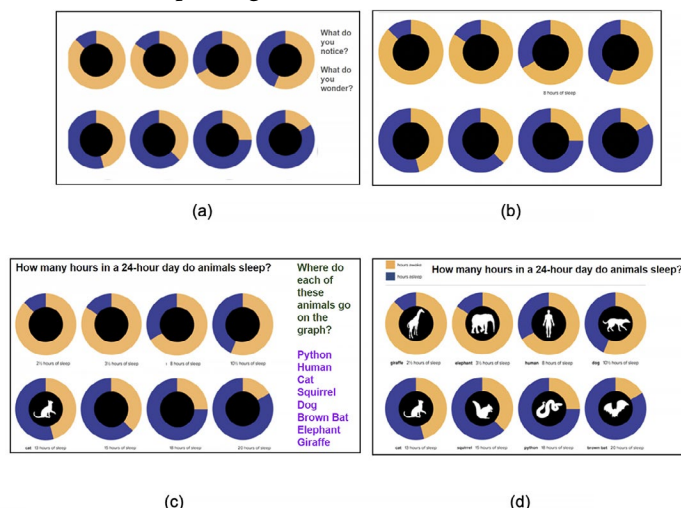
Graph-Talk Routines

In our workshop, we used two graph-talk routines—slow reveal graphs and data talks—to provide our participants with further opportunities to engage in nuanced discussion and reasoning about data represented in graphical form. The slow reveal graph (Laib, n.d.) routine begins with an entirely unlabeled graph on the first slide in a projected slideshow, and the facilitator advances through the slideshow to reveal more labels on the graph incrementally. Participants are engaged via reflective questions and discussion with every new reveal. As a more concise, graph-based routine, data talks (Boaler et al., 2021) involve the analysis and discussion of a complete graph projected onto a screen. In this routine, all details of the graph are revealed from the beginning, which allows for participants to notice, wonder, and discuss any features of the graph that catch their attention.

Slow Reveal Graphs

We used slow reveal graphs (Laib, n.d.) to generate intense focus on the structure, values, context, and story of graphs and data representations, helping participants pay particular attention to the connections to numbers and operations. The first step in a slow reveal is showing the graph without any context or number clues. An example created from a graph in the book, *Animals by the Numbers: A Book of Infographics* (Jenkins, 2016) and found on the website, “Slow Reveal Graphs” (Laib, n.d.), appears in Figure 5(a). Participants examined the structure of this “bare” graph and reflected on what they noticed and wondered.

Figure 5
Slow Reveal Graph Progression

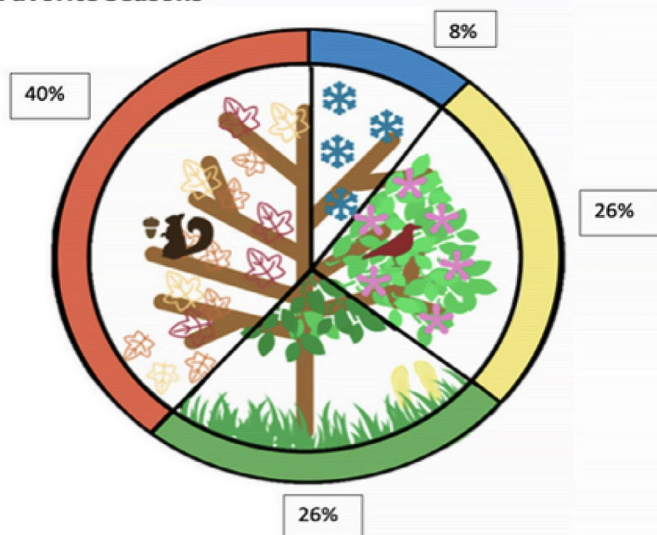


Note. These images depict a slow reveal graph progression (Laib, n.d.).

As a slow reveal graph routine continues, information is revealed bit by bit (see Figure 5b–5d). Each time new information was revealed, we asked participants to consider what they knew about the graph and its underlying data. In the example from our workshop that appears in Figure 5, it was first revealed that one circle was labeled “8 hours of sleep” (see Figure 5b). Then, we challenged participants to label all circle values using estimation and the proportionality of the graph. Next, the context of the graph was revealed via the graph’s title, “How Many Hours in a 24-Hour Day Do Animals Sleep?” The number of hours a squirrel sleeps was also revealed, along with the list of the other animals on the graph (see Figure 5c). Participants were then asked to guess where each animal might go on the graph. In doing so, participants shared their funds of knowledge about different animals and their characteristics that may indicate sleeping patterns. For the grand finale, we shared the entire graph (see Figure 5d). A chorus of “Oh!” and “I knew it!” with smiles and high-fives erupted around the room.

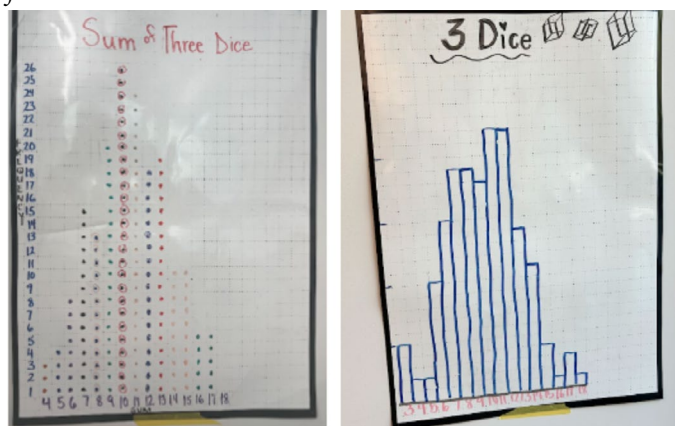
Data Talks

We also introduced the instructional routine of data talks (Boaler et al., 2021). Data talks begin with a graph of all information present from the outset (see Figure 6) and the questions, “What do you notice? What do you wonder?” During the workshop, participants compared data provided on the graph to their favorite seasons and wondered why so few people liked winter. A strong numerical comparison of data and connection to the graph context often emerged during our data talks. As the week progressed, we noticed a more sophisticated understanding.

Figure 6*Four Seasons Data Talks Example***Favorite Seasons**

Note. Values were included to support number sense and scale within the graph, provoking further data thoughts (Boaler, n.d.).

Initially, we used premade data representations for our data talks; however, many of our Data ICONS activities generated data and graphs of their own. For example, we collected data on the sum of three dice. Teams created a variety of data representations of their results, and we then used these graphs for data talks (see Figure 7).

Figure 7*Sums of Three Dice Graphs Created by Participants and Used for Data Talks*

Our data talks about participant-generated graphs led to deeper discussions on underlying reasons for the data collection results. For example, participants discussed which representation was more effective (i.e., dots or bars), the similarities and differences of the collected data, and why 11 and 12 were the most popular sums when rolling three dice.

CONCLUSION

Connections are at the heart of robust mathematical understanding. In her book, *Limitless Mind*, Boaler (2019) underscored the dire importance of every mathematics learner understanding mathematics as an interconnected subject rather than a series of isolated concepts to memorize. Boaler warned, without this concept of interconnectedness, teachers may inadvertently stifle students' ability to see the beauty and utility of mathematics in their lives. Moreover, Ma (1999) famously argued that a deep understanding of the interconnectedness of mathematical concepts is a vital part of teachers' knowledge. Only when teachers see the links between mathematical concepts can they plan math activities that position their students to discover those connections for themselves. Teachers must build a coherent and connected understanding of mathematics for their students rather than a fragmented one.

As we illustrated with the collections of activities described in this article (i.e., data blocks, datafied counting collections, and graph-talk routines), the topic of data yields a perfect foundation for discovering and experiencing the interconnectedness of mathematics. With data blocks, the values of data points can be compared, ordered, and grouped visually and conceptually. With datafied counting collections, collections and their counts become data for a graph or a number talk. With graph-talk routines, estimation and interpreting numbers in context become focal points for discussion and deep reasoning. With these activities (and others), teachers and their students alike can enjoy data-filled experiences throughout the academic year while deepening their understanding of all strands of the mathematics curricula.

REFERENCES

- Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., & Spangler, D. A. (2020). *Pre-K–12 guidelines for assessment and instruction in Statistics Education II (GAISE II): A framework for statistics and data science education*. National Council of Teachers of Mathematics. https://www.amstat.org/docs/default-source/amstat-documents/gaiseiiprek-12_full.pdf
- Boaler, J. (n.d.). *You cubed*. <https://www.youcubed.org/resource/data-talks/>
- Boaler, J. (2019). *Limitless mind. Learn, lead, and live without barriers*. HarperOne Publishers.
- Boaler, J., LaMar, T., & Williams, C. (2021). Making sense of a data-filled world. *Mathematics Teacher*, 114(7), 508–517. <https://doi.org/10.5951/MTLT.2021.0026>
- Bush, S. B., Karp, K. S., Albanese, J., & Dillon, F. (2014). The oldest person you've known. *Mathematics Teaching in the Middle School*, 20(5), 278–285. <https://doi.org/10.5951/mathteacmiddscho.20.5.0278>
- California Department of Education. (2023). *Mathematics framework*. <https://www.cde.ca.gov/ci/ma/cf/>
- Franke, M. L., Kazemi, E., & Turrou, A. C. (Eds.). (2018). *Choral counting and counting collections: Transforming the PreK–5 math classroom*. Stenhouse Publishers.
- Franklin, C. A., Kader, G. D., Bargagliotti, A. E., Scheaffer, R. L., Case, C. A., & Spangler, D. A. (2014). *Statistical education of teachers (SET)*. American Statistical Association. <https://www.amstat.org/docs/default-source/amstat-documents/edu-set.pdf>
- González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, 6(1–2), 115–132. https://doi.org/10.1207/S15327671ESPR0601-2_7
- Jenkins, S. (2016). *Animals by the numbers: A book of infographics* (S. Jenkins, Illus.). Clarion Books.
- Laib, J. (n.d.). *Slow reveal graphs*. <https://slowrevealgraphs.com/>
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States* (1st ed.). Routledge. <https://doi.org/10.4324/9781410602589>
- National Governors Association & Council of Chief State School Officers. (2010). *Common Core State Standards for mathematics*. Common Core State Standards Initiative. https://corestandards.org/wp-content/uploads/2023/09/math_standards1.pdf
- Parrish, S. (2010). *Number talks: Helping children build mental math and computation strategies, Grades K–5*. Math Solutions.