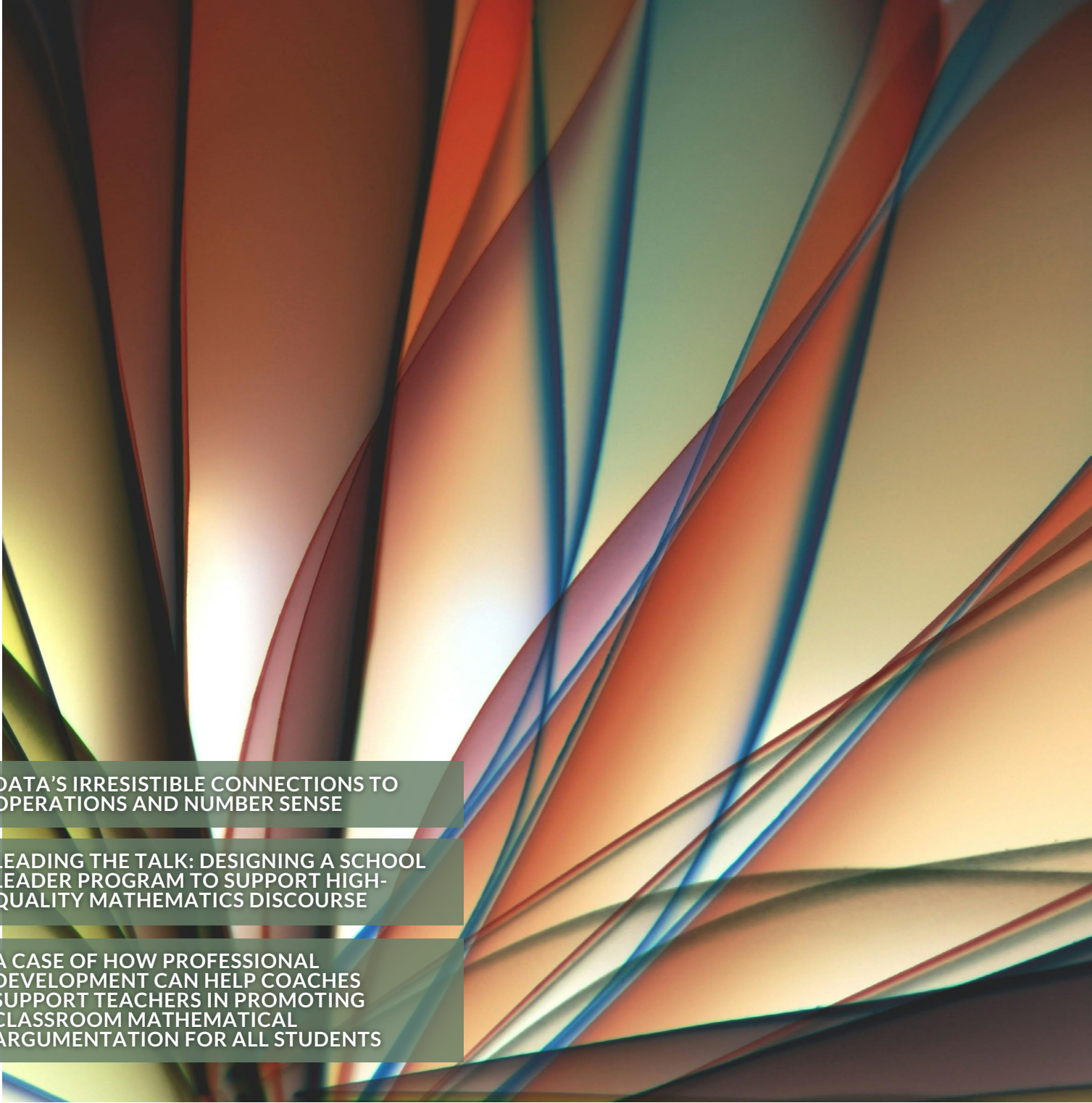


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DATA'S IRRESISTIBLE CONNECTIONS TO
OPERATIONS AND NUMBER SENSE

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LEADER PROGRAM TO SUPPORT HIGH-
QUALITY MATHEMATICS DISCOURSE

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This article discusses a workshop designed to enhance elementary teachers' understanding of how data, number sense, and operations are interconnected. Despite the important role of data in the Common Core State Standards, many teachers have reported feeling unprepared to teach data concepts. Accordingly, the workshop engaged teachers in hands-on activities (i.e., data blocks, datafied counting collections, graph-talk routines) designed to deepen their knowledge and confidence in teaching data. These activities helped teachers explore connections between data and other math concepts (e.g., comparison, counting, estimation). The article highlights the importance of viewing mathematics as an interconnected whole and offers strategies to integrate data-rich activities into classroom instruction, promoting deeper mathematical understanding and student engagement.

13 LEADING THE TALK: DESIGNING A SCHOOL LEADER PROGRAM TO SUPPORT HIGH-QUALITY MATHEMATICS DISCOURSE

by Zuhail Yilmaz, Paola Sztajn, Daniel J. Heck, Kristen A. Malzahn, Julia Schnegg, Lindsey H. Sachs, Edith Gummer, Francine P. Riddick

This article presents the design and evaluation of the Project All Included in Mathematics (Project AIM) School Leader Program (SLP), which supports principals in fostering high-quality mathematics discourse. We used design research to conduct three studies examining (a) SLP's design principles, (b) activity alignment with the National Council of Supervisors of Mathematics (NCSM) essential actions (EA) framework for leadership in mathematics education, and (c) principals' perspectives on SLP's contribution to their learning and leadership practices. Findings showed strong alignment between principals' needs and the design principles, consistent connections between SLP activities and EA, and key principals' perspectives, highlighting growth in instructional vision, capacity, and instructional support. Results also underscore the importance of aligning leader and teacher learning, embedding practical leadership tools, and approaching EAs as a guide when designing professional development (PD) for school leaders.

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by Anushree Bopardikar, Jennifer Knudsen, Christina B. Silva, Harriette S. Stevens

This article overviews how professional development (PD) can help coaches support teachers in fostering classroom mathematical argumentation (CMA) for all students. After summarizing conceptual underpinnings of a PD program on coaching, the article describes the program, presenting a case study of how structures and tools designed by the PD for lesson planning and reflection informed a coach's discourse with a teacher. Analyses of transcripts showed the extent to which a coach-teacher dyad discussed various topics related to mathematical concepts and practices and students' participation during early and late coaching cycles, informed by the purposes and protocols designed to visualize, plan, and debrief lesson enactment. The article highlights recommendations for PD design and research to support mathematics coaches.

50 INFORMATION FOR REVIEWERS

A CASE OF HOW PROFESSIONAL DEVELOPMENT CAN HELP COACHES SUPPORT TEACHERS IN PROMOTING CLASSROOM MATHEMATICAL ARGUMENTATION FOR ALL STUDENTS

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ABSTRACT

This article overviews how professional development (PD) can help coaches support teachers in fostering classroom mathematical argumentation (CMA) for all students. After summarizing the conceptual underpinnings of a PD program on coaching, the article describes the program, presenting a case study of how structures and tools designed by the PD for lesson planning and reflection informed a coach's discourse with a teacher. Analyses of transcripts showed the extent to which a coach and a teacher discussed various topics related to mathematical concepts and practices and students' participation during early and late coaching cycles, informed by the purposes and protocols designed to visualize, plan, and debrief lesson enactment. The article highlights recommendations for PD design and research to support mathematics coaches.

Keywords: professional development, coaching, mathematical argumentation, qualitative research, case study

Since the late 1980s, prominent professional organizations (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSSO], 2010) have proposed ambitious goals for all students' learning of mathematics. These goals prioritize conceptual understanding and procedural proficiency in mathematics, constructing mathematical arguments in learning mathematics, communicating mathematical ideas to others, and using a variety of representations (NCTM, 2000; NGA Center & CCSSO, 2010). These organizations also emphasize fostering a positive attitude and confidence in mathematics among students.

Furthermore, every student should have "access to rigorous mathematics that develops their understanding, skills and

knowledge, along with the confidence to leverage their learning" (National Council of Supervisors of Mathematics, 2024, para. 5) and improve their world. NCTM (2014) further emphasized teaching practices that facilitate classroom discourse and support students in analyzing and comparing mathematical arguments. Aligned with these student learning goals are various models of teachers' professional development (PD), including coaching, which research has shown can have positive effects on teaching practices and students' assessment scores (further discussed later in this paper), particularly with highly trained coaches (Campbell & Malkus, 2011). In fact, significant policy-related publications authored by NCTM (2020) have advocated for coaching as a means for improving student learning.

Although a coach's assigned duties may include managing assessments and other administrative tasks (Campbell & Griffin, 2017), key coaching activities take place in the teacher-coach dyad (i.e., two people who work together with a common purpose). This article presents a case of how a coach drew on innovative structures and tools designed by the Visualize Teaching Professional Development (VTPD) program to support a teacher in promoting classroom mathematical argumentation (CMA) for all students. The case study addressed the following research question: How do PD structures and tools designed for lesson planning and reflection inform the topics of a coach's discourse with a teacher in promoting classroom mathematical argumentation for all students?

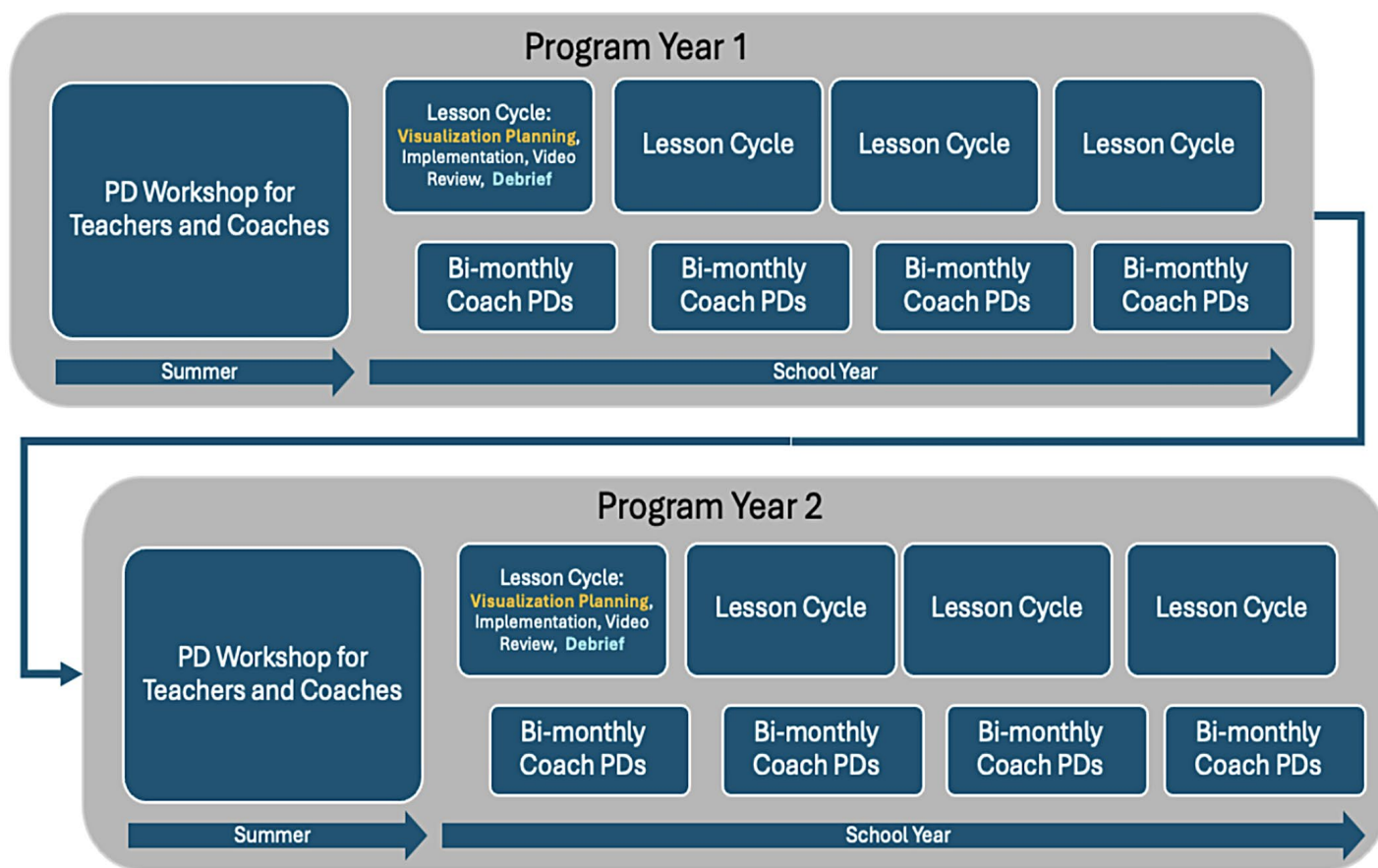
We describe the VTPD program in the next section, highlighting its conceptual underpinnings and associated structures and tools for coaches and teachers. This description is followed by a review of relevant literature on coaching. Next, the research methods are explained. The results show portions of the VTPD program in use in a coach's one-on-one practice with a teacher, with evidence highlighting how the coach took up structures and tools in a meaningful way in her interactions with the teacher. Finally, the article discusses limitations of the study and offers recommendations for PD design and research to support mathematics coaches.

Overview of the VTPD Program

The VTPD program focuses on teaching and learning mathematics argumentation, a high-level disciplinary practice that is important for all students to learn (Knudsen et al., 2018). Its goals are that teachers acquire (a) the ability

¹Anushree Bopardikar and Jennifer Knudsen are both first authors of this article.

Figure 1
Content Foundation: CMA



to make mathematical arguments for themselves and (b) a basic set of teaching moves to support every student in making their own arguments. The 2-year program includes (a) two 4-day summer workshops; (b) coach–teacher dyads engaging in the recommended number of four to six coaching cycles, each incorporating specially designed lessons, throughout 2 school years; and (c) monthly meetings with project staff to support coaches (see Figure 1). This study is situated in the coaching cycle.

Any math-focused coaching is influenced by consideration of what mathematics is important to learn (Martin & Harel, 1989); thus, the mathematics VTPD addresses is critical to the nature of what coaches and teachers do together. The major content focus of VTPD is mathematical argumentation in the classroom, which is emphasized in many current national and state standards (e.g., National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). VTPD is designed to support CMA, which is defined as a social process in which students and their teacher discover the mathematical truth together (Knudsen et al., 2018).

Over the first years of the VTPD team’s work with teachers, which preceded the current study, the team designed a four-part model of argumentation collaboratively with teachers new to this mathematical practice to support their own learning and use in their classrooms. The model includes stages of conjecturing (i.e., making bold claims based on evidence), justifying (i.e., establishing the truth or falsity of

conjectures), concluding (i.e., coming to common agreement on the truth status of the conjecture), and generating cases (i.e., finding examples to suggest patterns that can yield conjectures; see Figure 2). VTPD is grounded in prior research on the structure of mathematical argumentation (Harel & Sowder, 2007; Krummheuer, 1995). CMA and the four-part model are important to understanding the coach–teacher discourse examined in this study.

Figure 2
Four-Part Model of CMA

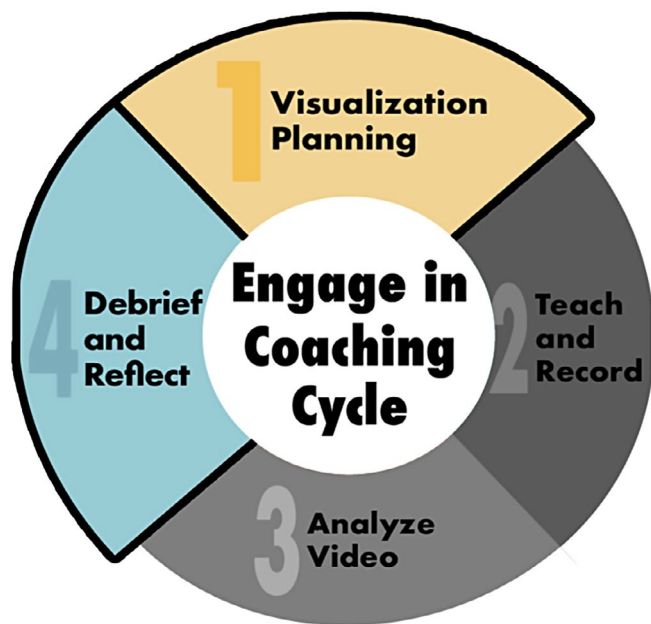


The Coaching Cycle

VTPD's coaching cycle consists of plan, teach, analyze, and debrief (or reflection) phases (see Figure 3 for overview of coaching cycle; Phases 1 and 4, as highlighted in this figure, were the foci of the study). Protocols, available for each phase of the four-part coaching cycle, guide coach–teacher dyads in structuring their time together.

Figure 3

Four-Phase Coaching Cycle



To prepare for visualization planning (VP), in Phase 1 of the cycle, the teacher selects a lesson designed to support CMA. Not every math lesson, even those that address conceptual understanding, affords students opportunities to make conjectures and justify them. VTPD offers focal lessons, some of which are based on lessons from Illustrative Mathematics (IM, 2021), an open-access curriculum available online to schools and teachers worldwide, and all of which address grade-level standards. The lessons are modified to create those affordances.

VP is designed to direct coaches' and teachers' attention to students' arguments and teaching moves that support them. Figures 4 and 5¹ show a protocol designed to structure coaching routines for VP. The teacher uses a combination of imagination and experience to envision how this lesson might go with her students as she talks through the lesson. First, the teacher and coach complete the mathematics of the chosen lesson together, making mathematical arguments for themselves, an important precursor to what comes next. Then, they map out several different routes students may take to make arguments. The teacher also plans to address the needs of target students who are chosen because they rarely participate in whole-class discussions. In the dyad, the coach is the active listener, probing the teacher with questions, offering insights, and writing a lesson plan as the visualization proceeds.

Figure 4

VP Protocol: Teachers

Visualization Planning: Teacher's Role

01

Prepare the focal lesson

Select one of the lessons developed by the Visualization Teaching team. Review the lesson and begin considering how you will emphasize math argumentation for student learning.

02

Meet with your coach

Schedule at least 30 minutes to 1 hour with your coach to review your prepared focal lesson, allowing time to "do the math together" before discussing how you will teach the lesson.

03

Do the math together

Before discussing your focal lesson, work through the math problem together. Take this time to consider the different ways your students may answer the math problem.

04

Engage in Visualization Planning

Imagine what teacher moves you will make and how students respond. Go through as many "what if" scenarios as possible. For example, if you ask one student to explain their idea, but the other students don't understand it. What will you do next?

05

Schedule your debrief

After your visualization planning, ensure you schedule your debrief within 1 week of teaching your lesson. Set aside another 30 minutes to 1 hour to discuss a specific segment of your recorded lesson, utilizing the Discussion Guide to structure your debrief.

Figure 5

VP Protocol: Coaches

Visualization Planning: Coach's Role

01

Review the lesson

Before meeting with your teacher, review the selected focal lesson to adequately prepare yourself for discussing teacher and student moves to support your teacher through the visualization planning.

02

Do the math together

Before discussing the focal lesson, work through the math problem together. Take this time to consider the different ways your teacher's students may answer the math problem.

03

Engage in Visualization Planning

Support your teacher in imagining their teacher moves and how they will support student learning. Ask clarifying or probing questions, ensuring your teacher considers several possible scenarios for student learning.

04

Take Notes

As you go through the Visualization Planning session, ensure you are taking notes and filling out the template to maintain a record of the teacher's moves and possible student responses.

05

Schedule your debrief

After visualization planning, ensure you and your teacher have scheduled the debrief within 1 week of teaching the lesson. Set aside another 30 minutes to 1 hour to discuss a specific segment of the recorded lesson, utilizing the Discussion Guide to structure your debrief.

¹Figures 4–6 and the protocols depicted in them were developed by the VTPD team.

In Phases 2 and 3, the teacher uses the lesson plan, devised during the VP session, to teach and record the lesson. This video recording of the lesson is then uploaded to a cloud-based system, offering teachers, coaches, and researchers a simpler way to view the recordings, to which the teacher and coach can add commentary directly. This recording is the basis for Phase 3. The teacher chooses a segment of the video for analysis, and then the teacher and the coach view and annotate video time points independently. These annotations become topics of coach–teacher dialogue in Phase 4.

In Phase 4 (i.e., debrief), the coach and teacher engage in evidence-based discourse facilitated by the discussion guide (see Figure 6). This guide is a rubric of characteristics of good teaching practice for mathematical argumentation, derived from prior literature. Three characteristics are (a) maintaining cognitive demand of questions, (b) orchestrating classroom discourse, and (c) ensuring access to mathematical concepts and practices for every student (e.g., Kuchemann & Hoyles, 2009; Otten et al., 2015; Selling, 2016). Teachers, with coach support, rate their teaching for the target lesson in each of these categories. The guide also provides characteristics of productive student discourse for teachers to rate for the same lesson (see Figure 6 for details regarding instructions on rating teaching and student moves) and setting goals for the next lesson within the coaching cycle. Certain levels may not be clearly distinguishable. Teachers may decide to use only one or two of the levels for a particular category.

Additional Learning Activities for Coaches

In addition to participating in the 4-day summer workshops with their teachers, coaches have other learning opportunities with VTPD (e.g., 2 coach-specific workshop days bracketing the 4-day workshops with teachers). These 2 days address areas of interest to coaches (e.g., characteristics of adult learners, how the VTPD coaching model fits with their current coaching practice). During the 4-day CMA workshops with their teachers, coaches fortify existing relationships and are given opportunities to work in teacher–coach dyads to focus more deeply on their learning and see each other as colearners of argumentation. During the academic year, coaches work with their teachers through multiple coaching cycles. Coaches, on their own, also participate in bimonthly 1.5-hour virtual professional learning community meetings with program staff and other VTPD coaches from around the United States. VTPD coach meetings generally consist of a short social check-in, a mathematical argumentation activity, and discussions of coaches' VP or debrief sessions with teachers to spark further dialogue.

Supporting Each and Every Student

The goal of reaching each and every student is core to contemporary teacher PD (Nasir & Cobb, 2002; NCTM, 2000, 2014). VTPD is designed to foster all students' participation in CMA through teachers' use of moves in the three categories from the discussion guide (i.e., cognitive demand, orchestration, access) along with insights they gain from the summer workshop. As students become more comfortable making "bold conjectures" (Knudsen et al., 2018) and putting together justifications that are convincing to themselves and

others, they can become more active participants in class and more engaged in learning.

Teachers and coaches have ongoing opportunities to learn to use mathematics argumentation to promote classroom discourse. As a basis for CMA, teachers can choose from VTPD argumentation-focused lessons aligned with the standards teachers must address and typical curricular sequences to ensure CMA is not an add-on. During coaching cycles, the protocols and structures described previously continue to emphasize full student participation in CMA. In the debrief phase, the discussion guide provides support to reflect on the success of those strategies through analyses of both teacher and student moves.

LITERATURE REVIEW

Coaching Defined

VTPD coaches may be called disciplinary (i.e., content) or instructional coaches. A content-specific approach to coaching emphasizes the "students' learning of a particular subject, and teachers' plans, strategies, and methods are discussed in that context" (Mudzimiri et al., 2014, p. 4). In this regard, the student's mathematical learning is the primary focus of the coaching session (Mudzimiri et al., 2014). In contrast, the instructional coach focuses on a "broad range of instructional issues, which might include classroom management, content enhancement, specific teaching practices, formative assessment" (Knight, 2009, p. 31).

For this article, we considered VTPD coaches as engaged in a third type of coaching that includes elements of the two coaching types named previously—math instructional coaching. We adopted the definition by Kane and Saclarides (2022): "A mathematical instructional coach is an educator who has deep mathematical pedagogical knowledge and who works closely with classroom teachers to improve student learning in mathematics" (p. 8). In the rest of this article, we refer to VTPD coaches as "math coaches" or simply "coaches."

What Coaches Need to Know

Researchers have identified broadly what mathematics coaches need to know. Common categories include (a) mathematics knowledge for teaching, (b) leadership skills, (c) interpersonal skills, (d) knowledge of when to use specific coaching activities, and (e) knowledge of the practices they guide teachers from and toward (Coburn & Russell, 2008; Gibbons & Cobb, 2016; Kane & Saclarides, 2022). However, a focus on coaches' knowledge of how to promote full student participation has rarely been addressed in research on coaching (Marshall & Buenrostro, 2021), an issue central to VTPD coaching tools and protocols. Nonetheless, there are resources available for practitioners (e.g., Bennett, 2014; Civil et al., 2019; Nasir & Cobb, 2002; Secada, 1994; van Es et al., 2022). A promising resource is Aguilar's (2020) book, which offers practical advice for coaches with full student participation as the centerpiece. The project team provided this book, and distillations of literature in its tools and resources, to VTPD coaches.

Figure 6
Debrief Meeting Discussion Guide

Student Moves

Math Practice 3 - Construct arguments and critique the reasoning of others

Conjecturing

Level 1	Level 3
Students make specific conjectures.	Students make generalized conjectures.
When comparing car speeds with lines of different slopes, students state, "The blue car goes faster."	Students say: "the steeper the line, the faster a car travels."
Students give the answer to a linear equation with one variable.	Students conjecture about the method for solving <i>all</i> linear equations with one variable.

Justifying

Level 1	Level 2	Level 3
In trying to justify, students simply repeat the conjecture, cite authorities such as the teacher or textbook, or rely on visual information.	Students use cases (empirical). Students use a finite number of cases to support a specific or generalized conjecture.	Students use generic cases or general properties to support a generalized conjecture.
To justify that the sum of any two even numbers is even, students may say that last year's teacher told them that.	Students conjecture that the sum of any two even numbers is even, Then try $2+4$, and $4+8$, and stop there because "they work."	Students show with counters that pairs of even numbers have no leftovers. They say, "If you combine two even numbers there won't be any leftovers, so the sum is even."

Math Practice 5 - Using appropriate tools strategically

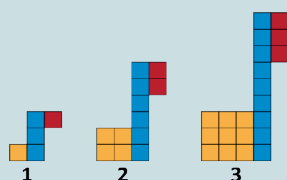
Level 1	Level 3
Students use tool procedurally under direct instruction by the teacher.	Students use a tool independent of direct instruction to express or explore an idea, generate cases, form conjecture, or make a justification
Students learn how to plot points on a coordinate graph following the teacher's instructions.	Students build a table to organize data or plot points in graphs to see patterns. Students use a shared diagram displayed by the teacher to communicate or clarify their conjecture/justification.

Math Practice 6 - Attend to precision

Level 1	Level 2	Level 3
Students or teacher ask for clarity or greater precision, AND	Students or teacher asks for clarity or greater precision, AND	Students or teacher asks for clarity or greater precision, AND
No attempt is made to further precision.	Students attempt to further precision but do not actually do so.	Students actually do further precision.
The teacher tells students their work is incorrect but offers no help or guidance, and the student does not try to correct it.	A student asks for clarification of a conjecture and a student re-words it in more precise mathematical language, but other students do not seem to understand the conjecture any better.	Students disagree with specific statements or imprecise calculations, and they offer clarification or correct calculations. Students refine a conjecture or explanation when someone says they don't understand it or don't think it addresses all cases and this helps the argument continue.

Math Practice 7 & 8 - Reason structurally

Level 1	Level 2	Level 3
Students focus on specific cases (of calculation, of solving an equation, of geometric shapes), without referring to patterns or general properties.	Students notice some patterns but without talking about the structure of those patterns (in calculations, expressions, and geometric shapes.)	Students focus on the relations between parts and the whole (i.e., structure) in number, expressions, or geometric shapes, or refer to general properties when conjecturing or justifying.



In finding a general pattern for any number n , students draw a shape for $n = 5$, and find the number blocks by counting individual blocks with no apparent systemic method.

Students say the pattern number is the same as the red blocks, but do not go further to see the effect on the overall growing pattern.

Students describe that blue blocks increase by 3 as the step number increases by 1 and expresses it generally, by saying “ $3n$ for any n .”

Students decompose a shape into rectangles and triangles and use their properties to find the area of the whole shape.

Teaching Moves

Maintenance of Cognitive Demand

If the teacher introduces a high-cognitive demand task, was the demand maintained?

Level 1	Level 2	Level 3
The teacher mostly asks questions so that students are mostly providing facts or procedures. Student turns are very short and directly respond to the teacher.	The teacher starts with a high-demand task, and starts out maintaining that demand, perhaps with a “why” question, but follow up questions mostly elicit facts or procedures only.	The teacher mostly asks questions that elicit student reasoning or engagement in the math practices and maintains this to the end of the task.
Asking “if we add these two numbers, what do we get?”	Asking, “Why does this algorithm work?” Then stating, “First, let’s make a rectangle. How should we divide it up?”	Asking students “why” questions that enable them to make their own justifications in support of a conjecture.

Orchestration

Does the teacher provide an opportunity for students to respond to each other’s ideas and interpret their peers’ ideas?

Level 1	Level 2	Level 3
The teacher rarely invites students to talk to each other. Discourse is mostly teacher-to-a-single-student.	The teacher encourages or invites students to respond to each other’s ideas, but often steps in to provide interpretation or additional questions between student turns. <i>Vertical discourse (t-s1-t-s2-t-s1)</i>	The teacher often encourages or invites students to respond to each other’s ideas, directly to each other.
<i>Vertical discourse (t-s-t)</i> Asking “OK, Darrel, how do you know your conjecture is true?” and sticking with Darrel.	“Thank you for your contribution. What do others think?”	<i>Horizontal discourse (t-s1-s2-s3-t)</i> Getting students to respond to each other with questions, answers, and offers.

Access

To what extent does the teacher support access to the task for each student, with particular attention to those who are typically left out?

Level 1	Level 2	Level 3
The teacher makes a broad invitation to all students to participate, but there is no follow up to engage students who are not part of classroom discourse.	The teacher makes an invitation to broaden participation in classroom discourse, but it is generic.	The teacher’s invitations to broaden participation are tailored to include those who have not yet been part of classroom discourse.
Asking “Who would like to make a conjecture?” and sticking with the one or two students who respond.	Asking “Does anyone else want to conjecture?”	Asking a student who is less confident in argumentation, “What do you think is false?” Then, “What do you think might be true?”

Progress Tracker

This is how you're doing on Teaching Moves and Student Moves

Cycle	Teaching Moves			Student Moves				
				MP3		MP5	MP6	MP7&8
	Cognitive Demand	Orchestration	Access	Conjecturing	Justifying	Strategic Tool Use	Attend to Precision	Reason Structurally
1								
2								
3								
4								
5								
6								

Reflections

Reflections on patterns of participation:

The Coaching Cycle

Many coaching programs are structured around a coaching cycle with at least three elements: planning, teaching, and reflecting on a lesson (Carey et al., 2011). Each element can be thought of as containing a set of routines. Gibbons and Okun (2023) asserted coaching routines can “manage the complexities of ambitious teaching collectively and raise questions or issues for further reflection” (p. 22). For example, Amador et al.’s (2023) study of coaching routines based on annotations of classroom videos found coaches’ practices shifted from summarizing aspects of the lesson to taking a more interpretative stance. Such research provided context for this study of the VTPD program, with an eye toward shifts in coaching practice to better support teachers.

Effectiveness of Coaching

A meta-analysis of 60 studies of coaching found significant improvement in a variety of teaching practices and measures of student achievement, including state assessments, National Assessment of Educational Progress scores, and researcher-made assessments (Harbour et al., 2018; Hill & Ball, 2004; Obara & Sloan, 2009). However, most of these studies were on literacy coaching, with only a few on mathematics coaching (Kraft et al., 2018). Two recent studies of large-scale mathematics coaching programs found significant positive effects on teachers’ classroom practices that promote students’ conceptual understanding and problem-solving skills (Kraft & Hill, 2020; Stein et al., 2022). These studies revealed models of mathematics coaching that are closely related to VTPD coaching. The studies, investigated, respectively, the planning and reflecting stages of a coaching cycle, the two parts investigated in this study.

First, Kraft and Hill (2020) studied the use of the Mathematical Quality of Instruction instrument in teacher-coach reflections/debriefs. Their study, particularly relevant to VTPD, suggested rubrics and classroom videos afford coaches’ and teachers’ opportunities to concentrate on important aspects of practice. Kraft and Hill posited that the resources gauge progress in these categories, with resulting changes in teachers’ classroom practices (e.g., giving mathematically correct explanations and other aspects of “ambitious teaching”).

Second, few research findings exist on how coaches develop their craft and what aids in doing so (Kane & Saclarides, 2022); however, using data collected in a Tennessee statewide coaching program, Stein et al. (2022) teased apart how coaches develop effective practices for lesson planning with teachers over time. The main study using the same data showed improved coaching practice in pre-lesson planning conversations (Russell et al., 2020) and improved teachers’ classroom practices.

METHODS

Participant Recruitment

To recruit participants for this PD program, our project team sought teachers and coaches in the United States who were already using IM in their curriculum. The use of IM ensured argumentation lessons provided by VTPD, which adapted IM lessons with more enhanced CMA content, fit into participants’ school-year curriculum and pacing. Another requirement to participate was the need for the teacher and the coach to commit to attending the workshop and complete all coaching cycles throughout the academic year. Though preferred, the coach did not have to specialize in mathematics. Whereas this paper focuses on a specific coach-teacher dyad (described in detail in the following section), the VTPD team provided PD to three coaches and seven middle-school mathematics teachers during the 1st year of the program in the 2021–2022 academic year.

Case Study Participants: Coach Celia and Teacher Belle

Our team conducted a qualitative descriptive case study (Yin, 2014) of a coach’s practice as part of a larger study of how the VTPD program supports mathematics coaches, teachers, and students to engage in CMA. Coach Celia² was selected as a case because she worked intensively with a teacher to complete all phases of a coaching cycle and across six cycles, as recommended by VTPD. She thus showed high commitment to supporting the teacher to engage in VP, enactment, video analysis, and debrief of lessons for each coaching cycle.

At the time of this study, Celia was employed as an instructional coach by her district, coaching teachers at an intermediate school in a suburban city in the southern United States that served students in Grades 5 and 6. Approximately 48% of the students were from racial minority backgrounds, including those who were Hispanic, Black, Asian, or multiracial. She had 5 years of coaching experience. Her content areas included English language arts (ELA), mathematics, science, and social studies. She had a bachelor’s degree in K–6 education and was a nationally board-certified generalist in early childhood education.

For the 2021–2022 academic year, Celia coached one teacher, Belle, for the VTPD program. Belle held a bachelor’s degree in K–6 education and taught mathematics at the school where Celia worked. In the 2021–2022 academic year, Belle taught sixth-grade mathematics. At the start of the study, Celia had worked for three years and Belle had worked for two years at this school/district. Beyond the coach-teacher relationship, Celia and Belle were friends and noted their friendship was a benefit to their work as a dyad during their participation in this PD project because it helped them engage with constructive feedback as they progressed through the coaching cycles.

³All names of coaches, teachers, and students are pseudonyms.

Data Sources and Analysis

The cycles comprising the data set for this study began in September 2021 and ended in May 2022, yielding a complete data set about VP and debrief meetings. In contrast, the other two coaches during this period worked with three teachers each, but these coach–teacher pairs conducted the cycles only partially due to various school and personal constraints. The resulting data set from their cycles was incomplete, offering limited opportunities for our team to study the practices of these coaches over time. Coach Celia’s work with the teacher provided our research team with the most available data, so this coach was selected as a best case, consistent with recommendations for conducting case study research (Yin, 2014) and prior case study research on coach–teacher discourse (Saclarides, 2022).

To better understand the nature of coach–teacher discourse, this article reports key findings from VP and debrief meetings, “each” for Cycles 2 and 5. These cycles were analyzed because they involved focal lessons designed by VTPD (providing our team with necessary context to understand the coach–teacher discourse), and the 6-month-long interval between the cycles provided a sufficiently long time to detect changes in their discourse.⁴

Meeting transcripts comprised the data set for this study, providing early and late data points respectively about coach–teacher discourse. Transcripts were analyzed to understand the topics of the discourse, which revealed the discursive learning opportunities fostered by the coach as she used the structures and tools designed for these meetings. Topics of discourse are indicators of the quality of discourse (Lefstein et al., 2020; Saclarides, 2022).

One member of our team reviewed each transcript holistically to understand the discussion, segmenting the transcript based on topically related exchanges (e.g., materials used in the lesson, the teacher’s goals or strategies, and students’ ideas). The segmentation was discussed and finalized by consensus with another author (Saclarides, 2022; Sherin & Han, 2004). Subsequently, the two authors coded the transcripts independently based on detailed coding schemes developed separately for VP and debrief meetings (see Appendices A and B). The coding schemes were developed deductively and inductively. The deductive approach drew on prior literature on teacher PD (van Es & Sherin, 2010), coaching (Saclarides, 2022), and video-based reflection (Borowiec et al., 2022) to capture various topics (e.g., lesson content, students’ disciplinary thinking, and pedagogical moves) to promote students’ thinking. Topics emerging inductively included lesson setup and discourse moves to provide opportunities for every student to contribute to classroom discourse.

The coding schemes were consistent with the tools and structures designed for VP and debrief conversations, which emphasized discussing (a) concepts and practices while doing the mathematics of a lesson together, (b) teacher moves for access and orchestration to help every student participate, (c) student thinking about mathematical concepts and practices, and (d) cognitive demand of teacher moves. The two authors

first coded and discussed a pair of VP and debrief transcripts. Following this discussion, the authors coded a second pair of VP and debrief transcripts independently and compared their codes to establish interrater agreement. There was 80% agreement on the coded VP data and 87.5% agreement on the coded debrief data. Any discrepancies in coding were resolved through discussion until there was 100% agreement.

After coding each transcript, the number of segments per code was counted to report frequencies of the topics discussed during a cycle. To compare within and between cycles, the frequencies were divided by the total number of segments in the transcript to calculate percentages of the discussion devoted to various topics. The percentages do not add up to 100 because segments were assigned multiple codes when appropriate. Finally, we developed and revised a case study report after receiving feedback from the coach, a member-check measure important for establishing credibility of the findings and interpretations (Guba, 1981).

FINDINGS

The findings highlight various topics that emerged during the coach–teacher discourse, showing how the structures and tools designed by the VTPD program were taken up as the coach and teacher engaged collaboratively with VP and debrief of lessons during Cycles 2 and 5. They used the protocol designed for VP and the discussion guide designed for debrief. The findings about Cycle 2 meetings, followed by Cycle 5 meetings, provide a detailed narrative of the discourse from each meeting, including relevant transcript excerpts to illustrate the main points.

Cycle 2

The focal lesson for Cycle 2 aimed to help students recognize questions about “how many groups?” and involved division operations using diagrams, multiplication equations, and division equations to represent such questions. The argumentation goal was to help students justify conjectures involving division problems in which the divisor and the quotient were not whole numbers (i.e., using fraction models and equations).

Cycle 2 VP

The coach and teacher devoted approximately 35 minutes to this meeting, mostly about potential teacher moves and/or goals for mathematical practices, namely argumentation (6 times, 43%; see Table 1 [data were calculated based on a total of 14 segments and ordered by frequency]). This finding was consistent with the VP protocol (see Figures 4 and 5), which asked them to consider opportunities in the lesson for taking up mathematical practices. They also discussed fractions as they completed the mathematics of the lesson together (4 times, 29%) and considered potential teacher facilitation of student participation (4 times, 29%). They devoted less time to anticipating students’ ideas about the target concepts and mathematical practices (3 times, 21%) and to anticipating students’ participation in classroom discourse (2 times, 14%).

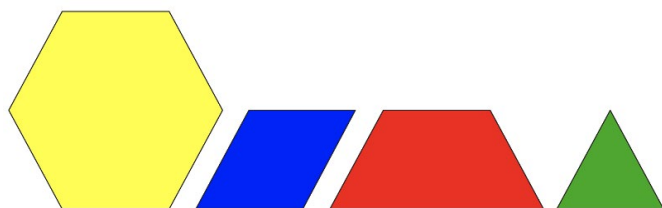
⁴The focal lessons for Cycles 2 and 5 are based on lessons from Illustrative Mathematics (IM, 2021).

Table 1
Percentages of Topics Discussed During Cycle 2 VP Meeting⁵

Topic	f	Discussion (%)
Planning teacher moves/goals for mathematical practices	6	43
Mathematical concepts of a lesson	4	29
Planning teacher facilitation of student participation	4	29
Anticipating student thinking about mathematical concepts	3	21
Anticipating student thinking about mathematical practices	3	21
Anticipating student participation in discourse	2	14
Planning teacher moves/goals for mathematical concepts	2	14
Lesson setup	2	14
Logistics	2	14
Mathematical practices of a lesson	1	7

The following excerpts illustrate how Celia and Belle considered potential teacher moves and/or goals to promote argumentation, which was the most prevalent topic of their discourse. Celia first reviewed a problem on fractions for different shapes, engaging the teacher in doing mathematics together, thereby laying a foundation to visualize and plan for specific pedagogical aspects of the lesson. Celia elicited Belle's thoughts about possible student understandings of the mathematics task and students' interpretations of the justifications presented in the task (see Figure 7). This flow of discourse was consistent with the VP protocol.

Figure 7
Diagram Showing Shapes the Students Discuss in This Lesson



Teacher: I just remember how hard this is for them to understand.
 Coach: What makes it hard?
 Teacher: Because a lot of them will agree with Diego [character in the mathematical problem].
 Coach: Because why?
 Teacher: [Referring to the uncovered part of the trapezoid] This is a third, and that's what's leftover.
 Coach: So, when they say one and a third, what is it that they're considering the whole?
 Teacher: There they're doing this as like an addition

problem. Does that make sense? . . . they think . . . rhombus plus a triangle gets me the trapezoid. They're not thinking rhombus plus what part of rhombus equals a trapezoid?

Coach: And what are they thinking?

Teacher: But instead, a lot of them will see . . . the missing piece is a triangle. So, rhombus plus triangle gets me trapezoid. Well, that's one rhombus plus a third . . . they mix multiplication [and] addition instead.

Finally, to enable Belle to consider potential goals and moves in helping students construct justifications and conclusions, Celia emphasized selecting and sequencing students to share their thinking. She asked clarifying and probing questions to help the teacher look for specific student reasoning during the whole-class discussion. Celia thus kept the discourse focused on teacher and student moves, consistent with the VP protocol.

Teacher: And so, once all teams [have] shared and kind of come up with a final solution, then I would want sharing to happen. So, then, as a whole group, we come to a conclusion.

Coach: Are you gonna select certain kids or look for certain things?

Teacher: It'd be interesting if I had a team that still supported Diego's.

Coach: What kind of reasoning would you look for? . . . For supporting Jada's thinking?

Teacher: Well, I definitely want to make sure that the people that are supporting it, use the rhombuses correctly. . . . They actually remember that's how many rhombuses are in that trapezoid . . . that whole idea of kind of taking the triangle off the table, if that makes sense. . . . And also, if there are students that come up with a multiplication or division equation, I would definitely want to illustrate that . . . "How did you get that?" "How does that support?" . . . I think one of the things about justification, too, . . . I know some students will choose to have diagrams or drawings, so maybe have . . . a variety of examples as well, like using the shapes, maybe drawing it out, having the equations, maybe if there are people that just have . . . written or verbal explanations.

Cycle 2 Debrief

During the debrief (approximately 24 minutes), there was a sharp focus on student participation. Celia and Belle commented most frequently on the teacher's facilitation of full student participation in the enacted lesson (3 times, 30%), on students' actual participation in the discourse (3 times, 30%) of the enacted lesson, and on students' thinking related to argumentation as observed in the enacted lesson (3 times, 30%). These data are included in Table 2, where calculations were based on 10 segments for topics and ordered by

⁵The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

frequency. About halfway through the meeting, they used the rubric from the discussion guide (see Figure 6) to rate various aspects of students' engagement with mathematical practices and the teacher's facilitation and to consider implications based on overall patterns of classroom discourse.

Table 2

Percentages of Topics Discussed During Cycle 2 Debrief Meeting⁶

Topic	f	Discussion (%)
Reflecting on teacher facilitation of student participation (in enacted lesson)	3	30
Reflecting on student participation in discourse (in enacted lesson)	3	30
Reflecting on student thinking about mathematical practices (in enacted lesson)	3	30
Reflecting on student thinking about mathematical concepts (in enacted lesson)	2	20
Planning teacher facilitation of student participation (for future lessons)	2	20
Anticipating student participation (for future lessons)	2	20
Reflecting on teacher facilitation of mathematical concepts (in enacted lesson)	1	10
Reflecting on teacher facilitation of mathematical practices (in enacted lesson)	1	10
Planning teacher moves/goals for mathematical practices (for future lessons)	1	10
Logistics	1	10

Some similarities and differences existed in the discourse of the two Cycle 2 meetings. The coach and teacher discussed the teacher's facilitation and students' thinking and participation during both meetings. These topics were broadly consistent with the purposes, tools, and structures of those meetings. However, they talked about students' actual ideas about mathematical practices and their actual participation in the debrief meeting rather than in the VP meeting, which was consistent with the debrief being designed to reflect on a lived experience of the enacted lesson and additionally on an annotated video of the lesson. Conversely, during debrief, they spent less time discussing the setup or logistics of future lessons and did not discuss the concepts or practices of the lesson. This finding contrasted with the VP meeting, where the latter topics were more pertinent and for which different tools and structures had been designed.

The coach and the teacher reflected together on moments of the enacted lesson they had previously annotated

independently on video. They focused on how students participated and how the teacher facilitated their participation with the help of specific goals, strategies, and structures (e.g., think-pair-share, individual think time), two of the most prevalent topics of the coach-teacher discourse during this debrief meeting. Celia used the discussion guide to help the teacher reflect on students' participation and on her facilitation of their participation in the enacted lesson (see Figure 6 for the rating scale about orchestration under "Teacher Moves"). In doing so, Celia commented on improvements in the teacher's facilitation (and in students' participation) and on how the teacher had used independent think time intentionally to promote more student-directed talk. As evidenced in the following excerpt, the coach also noted changes in students' inclinations to ask questions of their peers. When the teacher voiced a concern about facilitating participation of all students, hinting at a need to modify teaching practices for future lessons, the coach shifted away from talking about the enacted lesson and toward exploring instructional strategies for subsequent lessons to help the teacher improve her practice at orchestrating classroom discourse. In doing so, Celia invited Belle's perspective on specific matters, proposing a strategy to help students engage in questioning with their peers and prompting Belle to consider how she might group students:

- Coach:* I think students are starting to ask each other questions. . . . Because I think that kids are starting to, maybe we can think through how to . . . I don't know, it's been a natural progression so far, right?
- Teacher:* Yeah, I think it's the part of . . . when kids are off, how do I bring them? I think that's where I've got to grow the most.
- Coach:* Do you think it'd be helpful to have . . . questions for students to ask students to guide that discussion for that partner piece?
- Teacher:* But I think about . . . if I have two kids that are already below grade level, they don't know that they're in lost land any better than they're in the right land? Just to be honest.
- Coach:* Are your partnerships set up right now? In like—
- Teacher:* —intentional ways? No. I usually just partner them with whoever they're sitting by . . . rarely have them set up in a particular way because sometimes when we do partner work, I let them choose. Sometimes I just have them partner [with] the person beside them.
- Coach:* If they were intentional, how would you make them intentional?
- Teacher:* Well, my automatic response to that is to put . . . people that are about on the same level together because . . . I have some kids in this class that are . . . seventh-, eighth-grade math ability, I wouldn't put them with like any of my low kids because that's, the low kids will just give in to whatever they say. And so . . . it goes back to the discussions we've had in [professional learning community], where it's,

⁶The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

like, do we put like a 3 and a 5 together, and a 2 and a 4 together? But then . . . I always have this feeling that the upper kid will dominate. And be like, "No, that's dumb." Or like, "You're way off."

Coach: Yeah. So, . . . similar ability.

Cycle 5

The focal lesson for Cycle 5 aimed to help students understand that the surface area of a three-dimensional object with flat faces is a measurement of the combined area of all its faces. The argumentation goal of the lesson was to help students justify the generalizability of their methods for calculating surface area.

Cycle 5 VP

Approximately 36 minutes long, most of this VP meeting was about lesson setup, with Celia and Belle reviewing the lesson tasks, goals, and context (11 times, 74%). They did consider potential teacher moves and/or goals for supporting students' learning of mathematical concepts and practices (5 times, 33% each for concepts and practices), attending to some of the fine-grained aspects of mathematics teaching. Lesser time was devoted to anticipating student thinking. Table 3 provides data calculated based on 15 total segments and ordered by frequency.

Table 3

Percentages of Topics Discussed During Cycle 5 VP Meeting⁷

Topic	f	Discussion (%)
Lesson setup	11	74
Planning teacher moves/goals for mathematical concepts	5	33
Planning teacher moves/goals for mathematical practices	5	33
Anticipating student thinking about mathematical concepts	2	13
Anticipating student thinking about mathematical practices	2	13
Mathematical concepts of a lesson	2	13
Planning teacher facilitation of student participation	1	6

The following excerpt shows how Celia helped Belle consider her goals and moves for facilitating students' learning of mathematical concepts and of argumentation, two of the more prevalent topics during this meeting. After reviewing the lesson goals, mathematics problems, and content, they reviewed a math problem consisting of three parts. Part 1 asked students to find the surface area in square units of a rectangular prism that was 3 units high, 2 units wide, and 5 units long. Part 2 asked students to justify their method, and Part 3 asked students to state whether their method would work for any right rectangular prism. The following excerpt begins where Belle imagined engaging students in partner talk. Celia probed Belle to visualize in greater detail the

student ideas to which she might listen, encouraging Belle's choice of questions to ask during partner talk. In doing so, the coach helped the teacher not only consider her own moves but also attend to potential student thinking related to math practices, thus maintaining a dual focus on both pedagogy and student learning while planning toward curricular content, goals, and tasks:

Teacher: And then maybe we check in they could do partner work or a team check-in, of, what you get and . . . how'd you get there. . . . So, maybe with a partner, "What was your strategy? . . . What's your justification?" I think that I am . . . just kind of observing and just seeing . . . what they're getting.

Coach: Yeah, I like that idea of having those explicit questions out there too, though.

Teacher: And then after that, I don't know if we should have . . . everyone share: "Did we all get the same thing?" to make sure everyone got the same surface area.

Coach: What are you listening in for?

Teacher: Do they get the same answer? And do they have the same justification? Or justification that matches their answer?

Coach: So, while you're . . . actively monitoring, that's when you're listening in to see if they have the same answer.

Teacher: Slash [meaning "or"] correct answer, I guess.

Coach: Okay, and then you're also looking for their method of justifying. What . . . methods of justification do you think you'll see?

Teacher: So, I could see some people . . . counting the faces [of the prism]. So, . . . tactile, using the cube.

Belle considered how some students may count individual squares presented on the faces of the prism. Building on Belle's ideas, Celia helped her visualize how she could elicit students' ideas, as evidenced in the following excerpt:

Coach: So, after . . . you've given [students] the exit ticket, they worked on [Problem Parts] 1 and 2. They thought independently, they pair, then they go back independently to think through [Problem Part] 3. And to decide whether or not the strategy that they're using—the justification—will lead to a generalization where they could use it for any given rectangular prism. And then . . . when you say, "This is . . . what we are sharing," . . . how does that get shared?

Teacher: Well, . . . I guess we could do . . . a pair-share again. And then I think that it would be worth having a couple people share . . . whole group . . . patterns or generalizations.

⁷The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

Cycle 5 Debrief

The debrief meeting lasted approximately 21 minutes, during which Celia and Belle focused on mathematics argumentation and student participation. They commented most frequently on students' thinking related to mathematical practices in the enacted lesson (6 times, 43%), followed by commentary on Belle's facilitation of student participation (5 times, 36%). To a lesser extent, they discussed students' thinking about mathematical concepts (3 times, 21%) and how Belle supported students' understanding of mathematical practices (3 times, 21%). Celia helped Belle take stock of how students' learning and her facilitation were progressing toward her goal of participation by all students in CMA. In fact, in contrast to the Cycle 2 debrief meeting, the discussion guide was used early in this meeting after the coach had finished reviewing the lesson goals and the teacher's pedagogical goals and moves. Table 4 provides data calculated based on 14 segments for topics ordered by frequency.

Table 4
Percentages of Topics Discussed During Cycle 5 Debrief Meeting⁸

Topic	f	Discussion (%)
Reflecting on student thinking about mathematical practices (in enacted lesson)	6	43
Reflecting on teacher facilitation of student participation (in enacted lesson)	5	36
Reflecting on student thinking about mathematical concepts (in enacted lesson)	3	21
Reflecting on teacher moves/goals for mathematical practices (in enacted lesson)	3	21
Planning teacher facilitation of student participation (for future lesson)	3	21
Reflecting on teacher goals/moves for mathematical concepts (in enacted lesson)	2	14
Reflecting on student participation in discourse (in enacted lesson)	2	14
Revisiting curricular tasks/goals (in enacted lesson)	1	7

The following excerpt, in which the coach commented on Belle's ratings of her students' engagement with conjectures and justifications, demonstrates how Celia used the discussion guide to structure Belle's reflection on students' thinking about argumentation and her facilitation of student participation, which were among the more prevalent topics of the coach-teacher discourse (see Figure 6). The discussion guide helped them to better understand the different levels of justification (Knuth et al., 2009). Building on Belle's comments, Celia shared her own observations and interpretations about students' thinking, relating these back

to what they had anticipated previously, to help Belle assess her students' progress toward mathematical argumentation. In doing so, Celia added details about students' ideas and reasoning she had noted independently in the video of the enacted lesson:

- Coach:** *I thought Peter [a student] did an amazing job of communicating his thinking with how to do that. And . . . I was a little bit nervous when they started talking about length times width times height because I knew that volume was one of those areas that we had anticipated that they might have some misconceptions with. So, I thought it was really cool when they were able to . . . Leigh [a student] that was able to bring that up and make that connection at the end of like, how you could use the length and the height.*
- Teacher:** *So, I don't know if we want to do a 3 on that one [referring to rating students' conjecturing on the rating scale provided in the discussion guide]. And it's more of like, maybe like a 2.5. Okay. And then as far as justifying [referring to student moves for justification described in the discussion guide]. There's Level 1, Level 2, and Level 3.*
- Coach:** *I mean, I think for the kids that came up, they were able to . . . apply this to a general thing in order to, essentially, . . . put into words what the formula was before writing the formula.*

As shown in the following excerpt, the coach continued to share her observations and inferences about specific teacher moves, focusing on her orchestration of classroom discourse and her facilitation of student access, which were criteria highlighted in the discussion guide. In doing so, the coach posed questions to help the teacher ponder next steps, and she tailored suggestions to areas of improvement identified by the teacher:

- Teacher:** *And then, access [referring to the rating scale in the discussion guide].*
- Coach:** *I think that think-pair-share really helped with the access.*
- Teacher:** *And allowing them to have independent think time to . . . I know, for some of my . . . quieter students, or slower processors, whatever. They have some time to think, like, "Does this work?" And I was pretty impressed that most of them were able to put something on the paper. Because sometimes I feel like students will just sit there—*
- Coach:** *—and wait.*
- Teacher:** *Or they'll . . . immediately be, like, "I don't know what I'm doing." I think I see . . . that pattern more in my first class because they just don't feel so successful in math. And so, it*

⁸The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

was really exciting to see at least they were all trying, and . . . attempting something. So, let's see—don't necessarily feel like I called anyone specific that hadn't participated.

Coach: *How do you push for more either?*

Teacher: *It's hard for me because . . . in my mind, I'm like, "Oh, we'll just ask the kids that aren't sharing anything." But then . . . maybe they don't have anything to share.*

Coach: *So, when you're doing that monitoring piece, maybe you start by monitoring those kids that are typically required . . . to see if there's something they could do. For yeah, like start with them as . . . what you're looking for.*

The findings show the broad goals, setup, and mathematical concepts and practices of a lesson were more prevalent topics during VP than during debrief in Cycle 5. These topics were aligned with the purpose and support designed for VP. Furthermore, there was greater discourse about students' actual mathematical thinking and participation and about the teacher's actual facilitation of argumentation and participation of all students during debrief than during VP, as the coach helped the teacher use the discussion guide rating scales to consider students' progress in constructing arguments and the teachers' moves in orchestrating discourse and providing access for participation. Finally, the coach and teacher devoted more time to planning facilitation of participation of all students for a future lesson during debrief than during VP in Cycle 5, for which the rating scales about teacher moves (i.e., orchestration and access) in the discussion guide provided a clear basis to formulate goals and next steps.

Comparing Coaching Cycles 2 and 5

Across Cycles 2 and 5, the coach supported the teacher to attend to both mathematical argumentation and participation of all students. Celia helped Belle consider not only her goals and moves for mathematical argumentation and student participation but also the progress her students made in terms of their mathematical thinking and participation in argumentation. Celia drew on the meeting protocol designed by the VTPD program. She asked clarifying and probing questions and responded with suggestions, observations, and interpretations to engage the teacher in (a) doing mathematics together; (b) anticipating her students' ideas, experiences, and challenges; (c) envisioning teacher moves; and (d) reflecting on how teacher facilitation, student learning, and participation unfolded during an enacted lesson.

There were also key differences in the topics between the cycles (see Tables 5 and 6). Cycle 5 VP focused more on broad lesson setup and less on the fine-grained aspects

of their planning compared to Cycle 2 VP. Furthermore, compared to Cycle 2 VP, there was less time devoted in Cycle 5 VP to doing mathematics together (i.e., to working out the mathematical concepts and practices of the lesson). Finally, compared to Cycle 2 debrief, there was greater talk in Cycle 5 debrief about (a) how students were thinking, especially about mathematical practices of the enacted lesson; and (b) the teacher's moves and goals in facilitating mathematical concepts, practices, and participation of all students.

Table 5

Comparison of VP Meeting Topics During Cycles 2 and 5⁹

Topic	Discussion during Cycle 2 VP (%)	Discussion during Cycle 5 VP (%)
Planning teacher moves/goals for mathematical practices	43	33
Planning teacher moves/goals for mathematical concepts	14	33
Planning teacher facilitation of student participation	29	6
Discussing mathematical concepts of a lesson	29	13
Discussing mathematical practices of a lesson	7	0
Anticipating student thinking about mathematical concepts	21	13
Anticipating student thinking about mathematical practices	21	13
Anticipating student participation in discourse	14	0
Lesson setup	14	74
Logistics	14	0

⁹The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

Table 6*Comparison of Debrief Meeting Topics During Cycles 2 and 5¹⁰*

Topic	Discussion during Cycle 2 debrief (%)	Discussion during Cycle 5 debrief (%)
Reflecting on teacher facilitation of student participation in CMA (in enacted lesson)	30	36
Reflecting on teacher moves/goals for mathematical concepts (in enacted lesson)	0	14
Reflecting on teacher moves/goals for mathematical practices (in enacted lesson)	0	21
Reflecting on student thinking about mathematical practices (in enacted lesson)	30	43
Reflecting on student thinking about mathematical concepts (in enacted lesson)	20	21
Reflecting on student participation in discourse (in enacted lesson)	30	14
Anticipating student participation (for future lessons)	20	0
Planning teacher facilitation of student participation (for future lessons)	20	21
Planning teacher moves/goals for mathematical practices (for future lessons)	10	0

DISCUSSION

This study revealed how PD structures and tools designed for lesson visualization, planning, and debriefing inform the topics of a coach's discourse with a teacher in promoting CMA for all students. The topics of coach-teacher discourse shifted within and between phases of Cycles 2 and 5. The differential use of the discussion guide may explain the differences observed between debrief meetings of Cycles 2 and 5 with respect to the extent to which the discussion was devoted to students' thinking and the teacher's moves and goals in facilitating the lesson. The rubrics focused and structured reflection on how students were conjecturing and justifying their mathematical arguments and on how the teacher worked toward cognitively demanding questions, orchestration, and access in classroom discourse. These rubrics were used early in Cycle 5 debrief to structure coach-teacher reflection, whereas the rubrics were introduced after almost half the meeting in Cycle 2 debrief.

On the other hand, the VP meetings did not show this kind of growth over time (except planning for the teacher's facilitation of mathematical concepts, which was more frequent in Cycle 5 than Cycle 2). A possible reason for this result concerns differences in the focal lessons for the two cycles. The Cycle 5 lesson about finding the surface area of a rectangular prism involved a three-part mathematical argumentation task for which students needed to calculate the surface area and justify their method and its generalizability for any rectangular prism. In contrast, the Cycle 2 lesson was more straightforward because it did not require students to justify the generalizability of their reasoning. A more complex, multistep mathematical argumentation task in Cycle 5 may have led the coach and the teacher to devote more time to reviewing the setup of the lesson than in Cycle 2.

The VP phase provided a dedicated time for the coach and teacher to brainstorm alternative paths through which a mathematical argumentation task could unfold. This phase included templates to help the coach and teacher attend to mathematical concepts and practices, pedagogical goals, and teacher and student moves with an overarching emphasis on CMA for all students. Additionally, principles of colearning and collaboration undergird many content-focused coaching models (Campbell & Griffin, 2017). The VP phase offered a structured opportunity for the coach and teacher to do mathematics together in the context of specific lessons, working as equals to make sense of the lesson content and to generate and reinforce shared goals for teaching and learning.

There is a lack of research focused on the reflection phase of the coaching cycle (Saclarides, 2022). In the VTPD program, the reflection phase is broken into two parts: (a) analyzing and commenting on the video independently and (b) subsequently a debriefing based in part on the video analysis. The coach drew on a clear structure and tools provided by the PD to engage the teacher with various topics based on her own independent viewing of the classroom video and emergent interactions with the teacher. Saclarides's (2022)

¹⁰The percentages do not add up to 100 because the segments were assigned multiple codes as appropriate.

case study portrayed reflection sessions that went deep into student thinking and teaching moves, as did this case study. Saclarides noted not all coaches are as expert as the coach in her case study and, therefore, may need further opportunities for developing coaching skills. This PD program provided an example of what that support could look like.

Furthermore, the coach's uptake of the discussion guide for the debriefing was important because it drew the discussion to several issues. The coach and the teacher noted the cognitive demand of the teacher's questioning, students' engagement with the conjectures and justifications, and the teacher's orchestration of classroom discourse and provision of access to CMA. With these rubrics, the coach guided the teacher to not only consider her own facilitation but also to consider students' thinking and participation in important disciplinary practices. When discussing access, the coach and teacher noticed how more students were drawn into class participation, attributing this shift to moves the teacher had made using the discussion guide's term "invitations" to a broad group of students. Similarly, when discussing cognitive demand, the coach and the teacher focused on student participation when they noted how the teacher's use of open-ended questions led a student who "was often quiet" to make a substantial contribution to class discussion.

Across the two cycles, there was generally greater attention to students' thinking and participation during debrief than during VP. This finding was expected due to differences in the purposes and support designed for VP and debrief meetings. Debrief meetings centered on reflecting on the past, with classroom events that could be reviewed on video and were addressed in the discussion guide, whereas VP meetings focused on envisioning student actions. The discussion guide also served as a tool for the coach and teacher to explore next steps and set goals jointly, grounding their work in the lived experience of the classroom and reinforcing a mutual, collaborative relationship between coach and teacher.

Finally, in some mathematics coaching programs, the coaches select video segments and comment on the segments before the teacher does (Gregory et al., 2017). In the VTPD program, coaches and teachers comment on the video concurrently and separately, made possible by cloud-based technologies. This approach to using video may add to the collaborative nature of coach-teacher interactions.

Limitations of the Study

This study focused on the case of a coach's practice for supporting a teacher by taking up PD structures and tools designed for visualizing, planning, and debriefing of lessons. However, the findings do not reveal how a coach may support multiple teachers and how a coach's practice may shift during subsequent rounds of implementing the coaching cycles. Furthermore, whereas the study shone light on the discursive learning opportunities fostered by the coach, it did not analyze other important indicators of the quality of coach-

teacher discourse, such as its depth or the extent to which the discourse includes reasoning, evidence, or details to substantiate claims (Lefstein et al., 2020; Saclarides, 2022).

Recommendations for Research on PD for Coaches

This study focused on the coaching practice of a single coach working with a teacher during a year of participation in a PD program. Future researchers may examine shifts in a coach's practice over longer term participation in PD and in response to multiple teachers. For instance, what levels of depth are attained while reflecting collaboratively on students' thinking and participation? How does a coach tailor support in response to teachers' own understandings of mathematical practices and in response to their understandings of students' prior experiences with CMA? Furthermore, cross-case analyses may compare practices of other coaches in supporting CMA for all students.

Recommendations for Designing Coaching Programs

This study leads to a number of recommendations to help other designers in developing PD programs that include coaching. First, we recommend use of well-structured protocols and routines to guide coach-teacher dyads in productive discourse. Protocols and routines are foundational to current coaching designs (Gibbons & Okun, 2023; Kraft & Hill, 2020; Stein et al., 2022). At the broadest level, a handful of structures—all of which can be introduced in joint coach-teacher PD or in coach-only PD—are useful in the coach-teacher dyad to further productive discourse. However, there is a balance to strike between structures that may be overly constrictive and structures that may be too open to provide adequate support for productive discourse. In the VTPD program, the balance was tilted toward more highly structured tools and routines that still enabled flexible use. This tilt was evidenced by the small list of topics the dyad spent most of their time discussing during VP and debrief meetings.

VP is a core set of routines that can be adopted or adapted in other programs. Excerpts from Cycle 5 showed the routine is flexible in supporting teachers' top-of-mind concerns. We recommend PD designers consider the phases through which teachers may move in the routines they design to ensure the routines are flexible enough to support the coach-teacher dyad through those phases, which may not be where designers imagined they would begin.

The debrief phase of VTPD coaching cycle relies on a discussion guide that presents a set of relevant, research-based categories to help situate teachers' practice in that research. Such research-based categories can ground discussions of practice that not only analyze enactments of prior lessons but also can lead to consideration of future improvements in practice. Therefore, PD designers may wish to similarly distill and efficiently present research-based categories relevant to their aims.

CONCLUSION

Professional organizations like the National Council of Supervisors of Mathematics (2024) have long envisioned that coaches and emerging school leaders need the “understanding, knowledge and skills to continue their own personal growth . . . and further develop excellence in mathematics teaching” (para. 3). This article reported on a PD program that includes coaching, combining aspects of the familiar coaching cycle and some innovative elements to support (a) the coach and teacher to engage in colearning during the summer institute and do the mathematics together during year-round coaching cycles, (b) the process of VP to anticipate students’ thinking and teacher moves, (c) both coach and teacher independently reflecting on and annotating video of enacted lessons prior to joint debriefing, and (d) enacted lessons based on research-based criteria characterizing classroom discussions. The study also exemplifies how structures and tools of the PD program designed for VP and debrief inform the topics of a coach’s discourse with a teacher in promoting CMA for all students. This study’s recommendations are applicable to the work of other PD designers, researchers, and practitioners in this area, in the same way prior research has informed VTPD.

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Appendix A: Visualization Planning Coding Scheme

Main code and definition	Subcodes	Definitions of subcodes
Math content: Coach and teacher make sense of the lesson content as they do the math together.	Math content – concepts	The discussion focuses on math concepts targeted in the lesson task.
	Math content – practices	The discussion focuses on the math practices targeted in the lesson task. Indicate the practice while applying this code.
Planning teacher moves/goals: Fine-grained decisions and consideration of potential moves and personal pedagogical goals for supporting students' learning. The moves may go beyond verbal discourse (e.g., questions or statements a teacher might make) to include provision of specific resources.	Planning teacher moves/goals – math concepts	The discussion focuses on teacher moves/goals related to specific math concepts targeted in the lesson tasks.
	Planning teacher moves/goals – math practices	The discussion focuses on teacher moves/goals related to specific math practices targeted in the lesson tasks. Indicate the practice while applying this code.
Anticipating student thinking/moves about mathematical concepts and practices: Anticipate student ideas and reasoning, prior knowledge and experiences, potential challenges, or misconceptions about math content. Imagine student actions and responses, as recommended in the Visualization Planning template.	Anticipating student thinking/moves – math concepts	The discussion focuses on potential student actions and responses related to specific math concepts targeted in the lesson tasks.
	Anticipating student thinking/moves – math practices	The discussion focuses on potential student actions and responses related to specific math practices targeted in the lesson tasks. Indicate the practice while applying this code.
Planning for participation of all students: Consider opportunities, challenges, prior experiences with orchestration, and access for student participation in math discourse. These factors may include commentary about participation structures (e.g., small group, whole-class, partner work).	Planning for participation – teacher facilitation	The discussion focuses on teachers' prior experiences, anticipating their actions, personal pedagogical goals, concerns, challenges in orchestrating classroom discourse, and providing students with access to the discourse.
	Anticipating student participation	The discussion focuses on students' prior experiences, anticipating their actions, and challenges with participation in and access to classroom discourse.

Appendix B: Debrief Coding Scheme

Code	Definition
Student thinking/moves – math concepts [enacted lesson]	Discuss students’ emergent understanding, difficulties, preconceptions, etc., about math concepts.
Student thinking/moves – math practices [enacted lesson]	Discuss students’ emergent understanding, difficulties, preconceptions, etc., about math practices emphasized in the VTPD program.
Teacher moves/goals – math concepts [enacted lesson]	Discuss the teacher’s actions/goals for supporting students’ understanding of mathematical concepts. The pedagogical goals reflect the teacher’s personal intentions for supporting students’ learning, which may account for students’ specific needs, interests, etc., in contrast to more general curricular or lesson goals.
Teacher moves/goals – math practices [enacted lesson]	Discuss the teacher’s actions /goals for supporting students’ understanding of mathematical practices. The pedagogical goals reflect the teacher’s personal intentions for supporting students’ learning, which may account for students’ specific needs, interests, etc., in contrast to more general curricular/lesson goals. Commentary may refer to the construct of “cognitive demand,” emphasized in the Discussion Guide.
Student participation [enacted lesson]	Discuss opportunities, challenges, changes, or patterns observed in students’ participation in and access to classroom discussions. Commentary may refer to orchestration and/or access, as recommended in the Discussion Guide.
Teacher facilitation of student participation [enacted lesson]	Discuss opportunities, challenges, changes, or patterns observed in teacher’s orchestration and provision of access to classroom discussions.
Revisiting curricular tasks/goals	Discuss general lesson tasks/goals. These may be documented in the lesson plan.
Logistics	Discuss technology for videorecording/video analysis, coaching cycle schedule, etc.
Planning teacher moves/goals – math practices [for future lesson]	Discuss potential strategies or goals for supporting students’ understanding of math practices for subsequent lessons.
Anticipating student participation [for future lesson]	Discuss potential opportunities, challenges, needs, etc. about students’ participation for subsequent lessons.